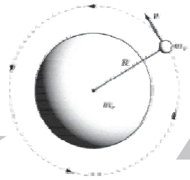


Unit - 6

Gravitation

6.1 Newton's Law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by $F \alpha \frac{m_1 m_2}{r^2}$.

or $$F = G\frac{m_1 m_2}{r^2}.$$

Also clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is Newton's third law of motion.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

(i) The value of G is 6.67×10^{-11} N-m^2 kg^{-2} in S.I and 6.67×10^{-8} dyne^{-} cm^2g^{-2} in C.G.S. system.

(ii) Dimensional formula $[M^{-1}L^3T^{-2}]$.

(iii) The value of G does not depend upon the nature and size of the bodies.

(iv) It does not depend upon the nature of the medium between the two bodies.

6.2 Acceleration Due to Gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g.

If M = mass of the earth and R = radius of the earth and g is the acceleration due to gravity, then

$$\therefore \qquad g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$$

(i) Its value depends upon the mass radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet.

(ii) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.

(iii) Dimension $[g] = [LT^{-2}]$

(iv) It's average value is taken to be 9.8 m/s^2 or 981 cm/sec^2, on the surface of the earth at mean sea level.

6.3 Variation in *g* with Height

Acceleration due to gravity at height h from the surface of the earth

$$g' = \frac{GM}{(R+h)^2}$$

Also $$g' = g\left(\frac{R}{R+h}\right)^2$$

$$= g\frac{R^2}{r^2} \qquad [\text{As } r = R + h]$$

(i) If $h << R$ $$g' = g\left[1 - \frac{2h}{R}\right]$$

(ii) If $h << R$. Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} \times 100\%$.

6.4 Variation in *g* with Depth

Acceleration due to gravity at depth d from the surface of the earth

$$g' = \frac{4}{3}\pi\rho G(R-d)$$

also g' $$= g\left[1 - \frac{d}{R}\right]$$

(i) The value of g decreases on going below the surface of the earth.

(ii) The acceleration due to gravity at the centre of earth becomes zero.

(iii) Percentage decrease $\frac{\Delta g}{g} \times 100\% = \frac{d}{R} \times 100\%$.

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

6.5 Gravitational Field

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational Field intensity : The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point. If a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then $\vec{I} = \frac{\vec{F}}{m}$.

6.6 Gravitational Potential

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point.

Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

Gravitational potential $V = -\frac{GM}{r}$

6.7 Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.

$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore \quad U = -\frac{GMm}{r}$$

If $r = \infty$ then it becomes zero (maximum).

6.8 Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

If v_e is the required escape velocity, then

$$v_e = \sqrt{\frac{2GM}{R}} \quad \Rightarrow \quad v_e = \sqrt{2gR}$$

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) For the earth, $v_e = 11.2$ km/sec

(iii) A planet will have atmosphere if the velocity of molecule in its atmosphere is lesser than escape velocity. This is why earth has atmosphere while moon has no atmosphere.

6.9 Kepler's laws of Planetary Motion

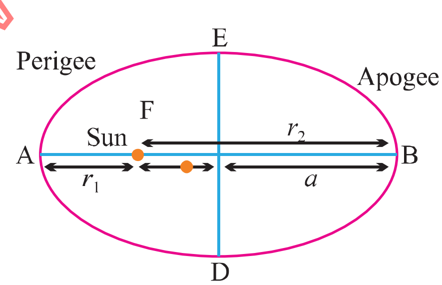
(1) The law of Orbits : Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

(2) The law of Area : The line joining the sun to the planet sweeps out equal areas in equal interval of time. *i.e.,* areal velocity is constant. According to this law planet will move slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.

Areal velocity $\frac{dA}{dt} = \frac{L}{2m}$

(3) The law of periods : The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.

$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$



where a = semi-major axis

r_1 = Shortest distance of planet from sun (perigee).

r_2 = Largest distance of planet from sun (apogee).

- Kepler's laws are valid for satellites also.

6.10 Orbital Velocity of Satellite

$$\Rightarrow \qquad v = \sqrt{\frac{GM}{r}} \qquad [r = R + h]$$

(i) Orbital velocity is independent of the mass of the orbiting body.

(ii) Orbital velocity depends on the mass of planet and radius of orbit.

(iii) Orbital velocity of the satellite when it revolves very close to the surface of the planet.

$$v = \sqrt{\frac{GM}{r}} = \sqrt{gR} \approx 8 \text{ km/sec}$$

6.11 Time Period of Satellite

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \qquad [\text{As } r = R + h]$$

(i) Time period is independent of the mass of orbiting body

(ii) $T^2 \propto r^3$ (Kepler's third law)

(iii) Time period of nearby satellite, $T = 2\theta\sqrt{\frac{R}{g}}$

For earth $\qquad T = 84.6$ minute ≈ 1.4 hr.

6.12 Height of Satellite

$$h = \left(\frac{T^2 gR^2}{4\pi^2}\right)^{1/3} - R$$

6.13 Geostationary Satellite

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth. The orbit of a geostationary satellite is known as the parking orbit.

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(ii) It sense of rotation should be same as that of earth.

(iii) Its period of revolution around the earth should be same as that of earth.

(iv) Height of geostationary satellite from the surface of earth $h = 6R = 36000$ km.

(v) Orbital velocity $v = 3.08$ km/sec.

(vi) Angular momentum of satellite depend on both the mass of orbiting and planet as well as the radius of orbit.

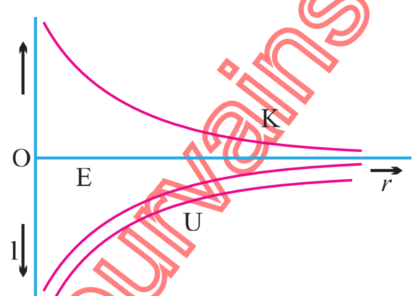
6.14 Energy of Satellite

(1) Potential energy : $U = mV = \dfrac{-GMm}{r} = \dfrac{-L^2}{mr^2}$

(2) Kinetic energy : $K = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2r} = \dfrac{L^2}{2mr^2}$

(3) Total energy : $E = U + K = \dfrac{-GMm}{r} + \dfrac{GMm}{2r} = \dfrac{-GMm}{2r} = -\dfrac{-L^2}{2mr^2}$

(4) Energy graph for a satellite



(5) Binding Energy : The energy required to remove the satellite its orbit to infinity is called Binding Energy of the system, *i.e.,*

$$\text{Binding Energy (B.E.)} = -E = \frac{GMm}{2r}$$

6.15 Weightlessness

The state of weightlessness (zero weight) can be observed in the following situations.

(1) When objects fall freely under gravity

(2) When a satellite revolves in its orbit around the earth

(3) When bodies are at null points in outer space. The zero gravity region is called null point.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. The mass of moon is nearly 10% of the mass of the earth. What will be the gravitational force of the earth on the moon, in comparison to the gravitational force of the moon on the earth ?

2. Why does one feel giddy while moving on a merry go round ?

3. Name two factors which determine whether a planet would have atmosphere or not.

4. The force of gravity due to earth on a body is proportional to its mass, then why does a heavy body not fall faster than a lighter body ?

5. The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside is zero, so can a body be shielded from gravitational influence ?

6. The gravitational force between two bodies in 1 N if the distance between them is doubled, what will be the force between them ?

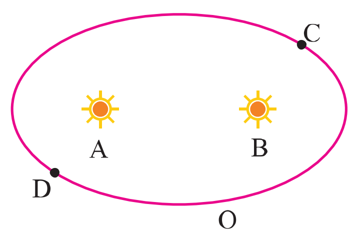
7. A body of mass 5 kg is taken to the centre of the earth. What will be its (i) mass, (ii) weight there.

8. Why is gravitational potential energy negative ?

9. A satellite revolves close to the surface of a planet. How is its orbital velocity related with escape velocity of that planet.

10. Two satellites A and B are orbiting around the earth in circular orbits of the same radius the mass of A is 16 times that of B. What is the ratio of the period of revolution of B to that of A ?

11. Identify the position of sun in the following diagram if the linear speed of the planet is greater at C than at D.



12. A satellite does not require any fuel to orbit the earth. Why ?

13. A satellite of small mass burns during its descent and not during ascent. Why ?

14. Is it possible to place an artificial satellite in an orbit so that it is always visible over New Delhi ?

15. If the density of a planet is doubled without any change in its radius, how does 'g' change on the planet.

16. Why is the weight of a body at the poles more than the weight at the equator ? Explain.

17. Why an astronaut in an orbiting space craft is not zero gravity although he is in weight lessness ?

18. Write one important use of (i) geostationary satellite, (*ii*) polar satellite.

19. A binary star system consists of two stars A and B which have time periods T_A and T_B, radius R_A and R_B and masses m_A and m_B which of the three quantities are same for the stars. Justify.

20. The time period of the satellite of the earth is 5 hr. If the separation between earth and satellite is increased to 4 times the previous value, then what will be the new time period of satellite.

21. Why does the earth impart the same acceleration to every bodies ?

22. If suddenly the gravitational force of attraction between earth and satellite become zero, what would happen to the satellite ?

Short Answer Type Questions (2 Marks)

23. If the radius of the earth were to decreases by 1%, keeping its mass same, how will the acceleration due to gravity change ?

24. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientation problem.

25. A satellite is moving round the earth with velocity v_0 what should be the minimum percentage increase in its velocity so that the satellite escapes.

26. The radii of two planets are R and 2R respectively and their densities ρ and ρ/2 respectively. What is the ratio of acceleration due to gravity at their surfaces ?

27. If earth has a mass 9 times and radius 4 times than that of a planet 'P'. Calculate the escape velocity at the planet 'P' if its value on earth is 11.2 kms^{-1}.

28. At what height from the surface of the earth will the value of 'g' be reduced by 36% of its value at the surface of earth.

29. At what depth is the value of 'g' same as at a height of 40 km from the surface of earth.

30. The mean orbital radius of the earth around the sun is 1.5×10^8 km. Calculate mass of the sun if $G = 6.67 \times 10^{-11}$ N m^2/kg^{-2} ?

31. Draw graphs showing the variation of acceleration due to gravity with (i) height above earth is surface (ii) depth below the earth's surface.

32. A rocket is fired from the earth towards the sun. At what point on its path is the gravitational force on the rocket zero ? Mass of sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. Orbital radius = 1.5×10^{11} m.

33. If the earth is one half its present distance from the sun. How many days will be presents one year on the surface of earth will change ?

34. A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth ?

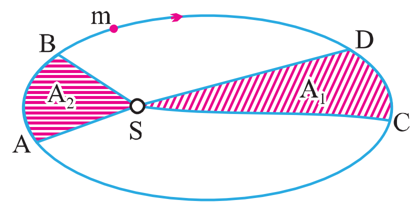
35. Why the space rockets are generally launched west to east ?

36. Explain why a tennis ball bounces higher on hills than in plane ?

37. The gravitational force on the earth due to the sun is greater than moon. However tidal effect due to the moon's pull is greater than the tidal effect due to sun. Why ?

38. The mass of moon is $\frac{M}{81}$ (where M is mass of earth). Find the distance of the point where the gravitational field due to earth and moon cancel each other. Given distance of moon from earth is 60 R, where R is radius of earth.

39. The figure shows elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from D to C and t_2, is time to move from A to B, what is the relation between t_1 and t_2 ?



40. Calculate the energy required to move a body of mass m from an orbit of radius 2R to 3R.

41. A man can jump 1.5 m high on earth. Calculate the height he may be able to jump on a planet whose density is one quarter that of the earth and whose radius is one third of the earth.

Short Answer Type Questions (3 Marks)

42. Define gravitational potential at a point in the gravitational field. Obtain a relation for it. What is the position at which it is (i) maximum (ii) minimum.

43. Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side. Also obtain the potential at the centre of the square.

44. Three mass points each of mass m are placed at the vertices of an equilateral triangle of side I. What is the gravitational field and potential at the centroid of the triangle due to the three masses.

45. Briefly explain the principle of launching an artificial satellite. Explain the use of multistage rockets in launching a satellite.

46. In a two stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the 2^{nd} stage gives it the necessary critical speed to put it in a circular orbit. Which stage requires more expenditure of fuel ? Given mass of earth = 6.0×10^{24} kg, radius of earth = 6400 km.

47. The escape velocity of a projectile on earth's surface is 11.2 kms^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth ? Ignore the presence of the sun and other planets.

48. A satellite orbits the earth at a height 'R' from the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence ?

49. Define gravitational potential. Give its SI units.

50. What do you mean by gravitational potential energy of a body ? Obtain an expression for it for a body of mass m lying at distance r from the centre of the earth.

51. What is the minimum energy required to launch a satellite of mass m kg from the earth's surface of radius R in a circular orbit at an altitude of 2R ?

Long Answer Type Questions (5 Marks)

52. What is acceleration due to gravity ?

Obtain relations to show how the value of 'g' changes with (i) altitude, (ii) depth.

53. Define escape velocity obtain an expression for escape velocity of a body from the surface of earth ? Does the escape velocity depend on (i) location from where it is projected (ii) the height of the location from where the body is launched.

54. State Kepler's three laws of planetary motion. Prove the second and third law. Name the physical quantities which remain constant during the planetary motion.

55. Derive expression for the orbital velocity of a satellite and its time period.

What is a geostationary satellite.Obtain the expression for the height of the geostationary satellite.

56. State and derive Kepler's law of periods (or harmonic law) for circular orbits.

57. A black hole is a body from whose surface nothing may ever escape. What is the condition for a uniform spherical mass M to be a black hole ? What should be the radius of such a black hole if its mass is the same as that of the earth ?

Numericals

58. The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30} kg. The mean distance of Jupiter from the Sun is 7.8×10^{11} m. Calculate gravitational force which sun exerts on Jupiter, and the speed of Jupiter.

59. A mass 'M' is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum.

60. If the radius of earth shrinks by 2%, mass remaining constant. How would the value of acceleration due to gravity change ?

61. A body released at the distance r ($r > R$) from the centre of the earth. What is the velocity of the body when it strikes the surface of the earth ?

62. How far away from the surface of earth does the acceleration due to gravity become 4% of its value on the surface of earth ? Radius of earth = 6400 km.

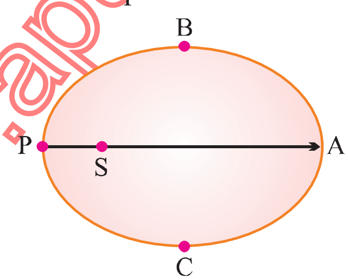
63. The gravitational field intensity at a point 10,000 km from the centre of the earth is 4.8 N kg^{-1}. Calculate gravitational potential at that point.

64. A geostationary satellite orbits the earth at a height of nearly 36000 km. What is the potential due to earth's gravity at the site of this satellite (take the potential energy at ∞ to be zero). Mass of earth is 6×10^{24} kg, radius of earth is 6400 km.

65. Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface, given that the escape velocity from the earth's surface is 11.2 km s^{-1}.

66. The distance of Neptune and Saturn from the sun is nearly 10^{13} m and 10^{12} m respectively. Assuming that they move in circular orbits, then what will be the ratio of their periods.

67. Let the speed of the planet at perihelion P in fig be v_p and Sun planet distance SP be r_p. Relate (r_A, v_A) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB ?



Answers For Very Short Questions (1 Mark)

1. Both forces will be equal in magnitude as gravitational force is a mutual force between the two bodies.

2. When moving in a merry go round, our weight appears to decrease when we move down and increases when we move up, this change in weight makes us feel giddy.

3. (i) Value of acceleration due to gravity

(ii) Surface temperature of planet.

4. $\therefore$ F = $\frac{GMm}{R^2}$, F $\propto m$ but $g = \frac{GM}{R^2}$ and does not depend on 'm' hence they bodies fall with same 'g'.

5. No, the gravitational force is independent of intervening medium.

6. F = 1 F′ = $\frac{F}{4} = \frac{1}{4}$N.

7. Mass does not change, weight at centre of earth will be 0 because $g = 0$.

8. Because it arises due to attractive force of gravitation.

9. $v_e = \sqrt{2}\ v_0$, $\therefore v_e = \sqrt{\frac{2GM}{R}}$ and $v_0 = \sqrt{\frac{GM}{R}}$ when r = R.

10. $T = \frac{2\pi r}{v}$ and $v = \sqrt{\frac{Gm}{r}}$, T is independent of mass, $\frac{T_B}{T_A} = 1 : 1 \Rightarrow T_A = T_B$.

11. Sun should be at B as speed of planet is greater when it is closer to sun.

12. The gravitational force between satellite and earth provides the necessary centripetal force for the satellite to orbit the earth.

13. The speed of satellite during descent is much larger than during ascent, and so heat produced is large.

14. No, A satellite will be always visible only if it revolves in the equatorial plane, but New Delhi does not lie in the region of equitorial plane.

15. 'g' gets doubled as $g \propto \rho$ (density).

16. As $g = GM/R^2$ and the value of R at the poles is less than that the equator, so g at poles is greater than that g at the equator. Now, $g_p > g_e$, hence $mg_p > g_e$ *i.e.,* the weight of a body at the poles is more than the weight at the equator.

17. The astronaut is in the gravitational field of the earth and experiences gravity. However, the gravity is used in providing necessary centripetal force, so is in a state of free fall towards the earth.

18. Geostationary satellite are used for tele communication and polar satellite for remote sensing.

19. Angular velocity of binary stars are same is $\omega_A = \omega_B$,

$$\frac{2\pi}{T_A} = \frac{2\pi}{T_B} \Rightarrow T_A = T_B$$

20. $\frac{T_2^2}{T_1^2} = \left(\frac{R_2}{R_1}\right)^3 \Rightarrow T_2^2 = 64 \times 25 \Rightarrow 40$ hr.

21. The force of gravitation exerted by the earth on a body of mass m is

$$F = G\frac{Mm}{R^2} = mg$$

Acceleration imparted to the body, $g = \frac{Gm}{R^2}$

Clearly, g does not depend on m. Hence the earth imparts same acceleration to all bodies.

22. The satellite will move tangentially to the original orbit with a velocity with which it was revolving.

Short Answers (2 Marks)

23. $g = \frac{Gm}{R^2}$ if R decreases by 1% it becomes $\frac{99}{100}R$

$$g' = \frac{GM}{(.99R)^2} = 1.02\frac{Gm}{R^2} = (1+0.02)\frac{Gm}{R^2}$$

$\therefore$ g' increases by $0.02\frac{Gm}{R^2}$, therefore increases by 2%.

24. (b), (c) and (d) are affected in space.

25. The maximum orbital velocity of a satellite orbiting near its surface is

$$v_0 = \sqrt{gR} = \frac{v_e}{\sqrt{2}}$$

For the satellite to escape gravitational pull the velocity must become v_e

But $v_e = \sqrt{2}v_0 = 1.414v_0 = (1+0.414)v_0$

This means that it has to increases 0.414 in 1 or 41.4%.

$\therefore$ The minimum increment is required, as the velocity of satellite is maximum when it is near the earth.

26. Here

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\cdot\frac{4}{3}\pi R^3 \rho$$

or $$g \propto R\rho$$

$\therefore$ $$\frac{g_1}{g_2} = \frac{R\rho}{2R.\frac{\rho}{2}} = 1:1.$$

27. $$v_e = \sqrt{\frac{2GM}{R_e}},\quad v_p = \frac{\sqrt{2GM_p}}{R_p}$$

$$M_p = \frac{M}{9}, R_p = \frac{R_e}{4}$$

$\therefore$ $$v_p = \sqrt{2G\frac{M}{9}\times\frac{4}{R_e}}$$

$$= \frac{2}{3}\sqrt{\frac{2GM}{R_e}} = \frac{2}{3}\times 11.2 = \frac{22.4}{3}$$

$$= 7.47 \text{ km/sec.}$$

28. $$g' = 64\% \text{ of } g = \frac{64}{100}g$$

$$g' = g\frac{R^2}{(R+h)^2} = \frac{64}{100}g$$

$\therefore$ $$\frac{R}{R+h} = \frac{8}{10}$$

$$h = \frac{R}{4} = 1600 \text{ km.}$$

29. $$g_d = g_h$$

$$g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$$

$$d = 2h = 2 \times 40 = 80 \text{ km.}$$

30. $R = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

$T = 365$ days $= 365 \times 24 \times 3600$ s

Centripetal force = gravitational force

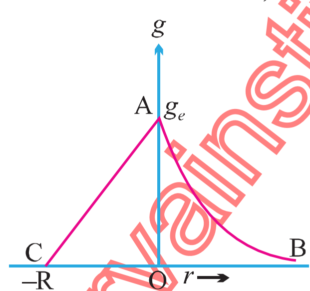
$$\frac{Mv^2}{R} = \frac{GMm}{R^2} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{GMm}{R^2}$$

$$M_s = \frac{4\pi^2 R^3}{GT^2}$$

$$= \frac{4 \times 9.87 \times (1.5 \times 10^{11})^3}{6.64 \times 10^{-11} \times (365 \times 24 \times 3600)^2}$$

$$M_s = 2.01 \times 10^{30} \text{ g.}$$

31. $g \propto \frac{1}{r^2}$ for $r > 0$ above surface of earth *i.e.,* AB



$g \propto (R - d)$ for $r < 0$ below surface of earth *i.e.,* AC

g is max. for $r = 0$ on surface.

32. Given $M_s = 2 \times 10^{30}$ kg,

$M_e = 6 \times 10^{24}$ kg, $r = 1.5 \times 10^{11}$ m

Let m be the mass of the rocket. Let at distance x from the earth, the gravitational force on the rocket be zero.

Then at this distance, Gravitational pull of the earth on the rocket

= Gravitational pull of the sun on the rocket.

i.e., $$\frac{GM_e m}{x^2} = \frac{GM_e m}{(r-x)^2} \text{ or } \frac{(r-x)^2}{x^2} = \frac{M_s}{M_e}$$

or $$\frac{R-x}{x} = \sqrt{\frac{M_s}{M_e}} = \sqrt{\frac{2 \times 10^{30}}{6 \times 10^{24}}} = \frac{10^3}{\sqrt{3}} = 577.35$$

or $$r - x = 577.35x$$

or $578.35\,x = r = 1.5 \times 10^{11}$

or $$x = \frac{1.5\times10^{11}}{578.35} = 2.59\times10^{8} m.$$

33. $T_1 = 365$ days; $r_1 = r$, $T_2 = ?$, $r_2 = r/2$

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

or $$T_2 = T_1.\left[\frac{r_2}{r_1}\right]^{3/2}$$

$$= 365\left[\frac{r/2}{r}\right]^{3/2} = 129 \text{ days}$$

Therefore decrease in number of days in one year will be

$$=365 - 129 = 236 \text{ days.}$$

34. Here $mg = 63$ N, $h = R/2$

As $$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+\frac{R}{2}}\right) = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$g_h = \frac{4}{9}g$$

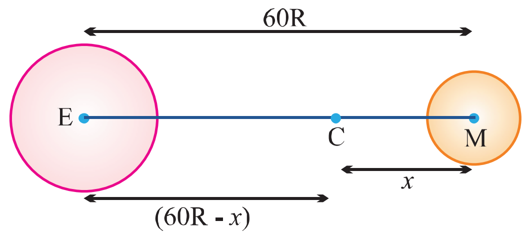
$$\therefore \quad mg_h = \frac{4}{9}mg = \frac{4}{9}\times63 = 28 \text{ N.}$$

35. Since the earth revolves from west to east, so when the rocket is launched from west to east the relative velocity of the rocket increases which helps it to rise without much consumption of fuel.

36. The value of 'g' on hills is less than at the plane, so the weight of tennis ball on the hills is lesser force than at planes that is why the earth attract the ball on hills with lesser force than at planes. Hence the ball bounces higher.

37. The tidal effect depends inversely on the cube of the distance, while gravitational force depends on the square of the distance.

38.



Gravitational field at C due to earth

$$= \text{Gravitational field at C due to earth moon}$$

$$\frac{GM}{(60R - x)^2} = \frac{GM/81}{x^2}$$

$$81x^2 = (60\ R - x)^2$$

$$9x = 60\ R - x$$

$$x = 6\ R.$$

39. According to Kepler's II[nd] law, area velocity for the palnet is constant

$$\therefore \quad \frac{A_1}{t_1} = \frac{A_2}{t_2},\ A_1 = 2A_2$$

$$\therefore \quad \frac{2A_2}{t_1} = \frac{A_2}{t_2}$$

$$t_1 = 2t_2.$$

40. Gravitational P.E. of mass m in orbit of radius R = U = $-\frac{GMm}{R}$

$$\therefore \quad U_i = -\frac{GMm}{2R}$$

$$U_f = -\frac{GMm}{3R}$$

$$\Delta U = U_f - U_i = GMm\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{GMm}{6R}.$$

41. $$g = \frac{4}{3}\pi GR\rho$$

$$g' = \frac{4}{3}\pi G R \rho'$$

The gain in P.E. at the highest point will be same in both cases. Hence

$$mg'h' = mgh$$

$$g' = \frac{mgh}{mg} = \frac{m \times \frac{4}{3}\pi G R \rho h}{m \frac{4}{3}\pi G R \rho'}$$

$$= \frac{R\rho h}{R'\rho'} = \frac{3R' \times 4\rho' \times 1.5}{R' \times \rho'}$$

$$= 18 \text{ m.}$$

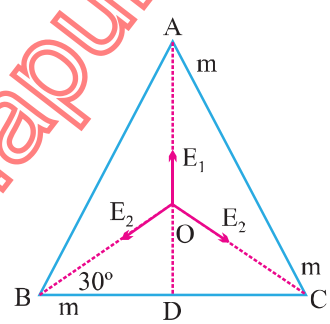
Answers For 3 Marks Questions

44.

$$E_1 = \frac{GM}{(OA)^2}$$

$$E_2 = \frac{GM}{(OB)^2}$$

$$E_3 = \frac{GM}{(OC)^2}$$



From Δ ODB, $\cos 30^\circ = \frac{BD}{OB} = \frac{l/2}{OB}$

$$OB = \frac{l/2}{\cos 30^\circ} = \frac{BD}{OB} = \frac{l}{2}\frac{2}{\sqrt{3}} = l/\sqrt{3}$$

Gravitational field at O due to *m* at A, B and C is say $\vec{E}_1, \vec{E}_2$ and $\vec{E}_3$

$$E = \sqrt{E_2^2 + E_3^2 + 2E_2E_3\cos 120^\circ}$$

$$= \sqrt{\frac{(GM3)^2}{I^2} + \left(\frac{3Gm}{I}\right)^2 + 2\left(\frac{3GM}{I}\right)\left(-\frac{1}{2}\right)}$$

$$= \frac{3GM}{I} = \text{along OD}$$

$\vec{E}$ is equal and opposite to $\vec{E}_1$

$\therefore$ net gravitational field = zero

As gravitational potential is scalar

$$V = V_1 + V_2 + V_3$$

$$= \frac{GM}{OA} - \frac{GM}{OB} - \frac{GM}{OC}$$

$$V = -\frac{3GM}{I/\sqrt{3}} = -3\sqrt{3}\frac{Gm}{I}.$$

46. Work done on satellite in first stage = W_1 = PE at 150 km – PE at the surface

$$W_1 = \frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMmh}{R(R+h)}$$

Work done on satellite in 2nd stage = W_2

= energy required to give orbital velocity v_0

$$= \frac{1}{2}mv_0^2 = \frac{1}{2}\left(\frac{GMm}{R+h}\right)$$

$$\frac{W_1}{W_2} = \frac{2h}{R} = \frac{2 \times 150}{6400} = \frac{3}{64} < 1$$

$W_2 > W_1$, so second stage requires more energy.

47. $V_e = 11.2\ \text{kms}^{-1}$, velocity of projection = $v = 3v_e$ Let m be the mass of projectile and v_0 the velocity after it escapes gravitational pull.

By law of conservation of energy

$$=\frac{1}{2}mv_0^2=\frac{1}{2}mv^2-\frac{1}{2}mv_e^2$$

$$v_0 =\sqrt{v^2-v_e^2}=\sqrt{9v_e^2-v_e^2}=\sqrt{8v_e^2}$$

$$=22.4\sqrt{2}$$

$$=31.68 \text{ km s}^{-1}.$$

48. The energy required to pull the satellite from earth influence should be equal to the total energy with which it is revolving around the earth.

The K.E. of satellite $=\frac{1}{2}mv^2=\frac{1}{2}m\frac{GM}{R+h}, \quad \therefore v=\sqrt{\frac{GM}{R+h}}$

The P.E. of satellite $=-\frac{GMm}{R+h}$

$$\therefore \quad \text{T.E.} = \frac{1}{2}\frac{mGM}{(R+h)}-\frac{GMm}{(R+h)}=-\frac{1}{2}\frac{GMm}{(R+h)}$$

$\therefore$ Energy required will be $\left(+\frac{1}{2}\frac{GMm}{(R+h)}\right)$.

51. $E_1=-\frac{GMm}{R}=-\frac{mgR^2}{R}=-mgR$

If v is velocity of the satellite at distance 2R, than total energy

$$E_2 = \text{K.E.} + \text{P.E.}$$

$$=\frac{1}{2}mv^2-\frac{GMm}{(2R+R)}$$

Orbital velocity of satellite, $v=\sqrt{\frac{GM}{2R+R}}$ or $v^2=\frac{GM}{3R}$

So, $\frac{1}{2}mv^2=\frac{GMm}{6R}$

$$E_2=\frac{GMm}{6R}-\frac{GMm}{3R}=-\frac{GMm}{6R}=-\frac{mgR}{6}$$

Minimum energy required to launch the satellite is

$$=E_2-E_1=-\frac{1}{6}mgR+mgR=\frac{5}{6}mgR.$$

Answers For Numericals

58. $$F = \frac{GMm}{r^2}$$

$$= \frac{6.67\times10^{-11}\times1.99\times10^{30}\times1.9\times10^{27}}{(7.8\times10^{11})^2}$$

$$F = 4.1 \times 10^{23} \text{ N}$$

$\therefore$ $$F = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{GMm}{r^2}\times\frac{r}{m}}$$

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{6.67\times10^{-11}\times1.9\times10^{30}}{7.8\times10^{11}}}$$

$$v = 1.3 \times 10^4 \text{ ms}^{-1}.$$

59. Let $m_1 = m$ then $m_2 = M - m$

Force between them when they are separated by distance 'r'

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}(Mm - m^2)$$

For F to be maximum, differentiate F w.r.t. m and equate to zero

$$\frac{dF}{dm} = \frac{G}{r^2}(M-2m) = 0$$

$$M = 2m;\ m = \frac{M}{2}$$

$\therefore$ $$m_1 = m_2 = \frac{M}{2}.$$

60. $$g = \frac{Gm^2}{R^2}$$

Taking logarithm $\quad \log g = \log G + 2\log m - 2\log R$

Differentiating it

$$\frac{dg}{g} = 0+0-2\frac{dR}{R} = -2\frac{dR}{R} = -2\left[\frac{-2}{100}\right]$$

$$\frac{dg}{g}\times 100 = -2\left[\frac{-2}{100}\right]\times 100 = 4\%.$$

61. Total Energy of the body $= \text{KE} + \text{PE} = 0 + \left[-\frac{GMm}{r^2}\right] = -\frac{mgR^2}{r}$

Let v be velocity acquired by body on reaching the surface of earth.

Total Energy on the surface$= \frac{1}{2}mv^2 + \left[-\frac{mgR^2}{R}\right] = \frac{1}{2}mv^2 - mgR$

According to law of conservatives of energy

$$\frac{1}{2}mv^2 - mgR = \frac{mgR^2}{r}$$

$$v^2 = 2gR - \frac{2gR^2}{r} = 2gR^2\left[\frac{1}{R} - \frac{1}{r}\right]$$

$$\Rightarrow \qquad v = R\sqrt{2g\left(\frac{1}{R}-\frac{1}{r}\right)}.$$

62. $g' = 4\%$ of $g = \frac{4}{100}\cdot g$

$$\frac{4}{100}g = g\left[\frac{R}{R+h}\right]^2$$

$$\frac{2}{10} = \frac{R}{R+h}$$

$$\therefore \qquad h = 4R = 4\times 6400 = 25{,}600 \text{ km}.$$

63. Gravitational intensity $= E = \frac{GM}{R^2}$

Gravitational potential $V = -\frac{GM}{R}$

$$\therefore \qquad \frac{V}{E} = -R$$

or $$V = -E \times R$$

or $$V = -4.8 \times 10{,}000 \times 10^3 = -4.8 \times 10^7 \text{ J kg}^{-1}.$$

64. $$U = \text{Potential at height } h = -\frac{GM}{R+h}$$

$$U = \frac{6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^6+36\times10^6} = -9.44\times10^6 \text{ J/kg}$$

65. Escape velocity from the earth's surface is

$$v_e = \sqrt{\frac{2GM}{R}} = 11.2 \text{ kms}^{-1}$$

Escape velocity from Jupiter's surface will be

$$v_e' = \sqrt{\frac{2GM'}{R'}}$$

But M′ = 318 M, R′ = 11.2 R

$$\therefore \qquad v_e' = \sqrt{\frac{2G(318M)}{11.2}} = \sqrt{\frac{2GM}{R}\times\frac{318}{11.2}}$$

$$= v_e'\times\sqrt{\frac{318}{11.2}} = 11.2\times\sqrt{\frac{318}{11.2}} = 59.7 \text{ kms}^{-1}.$$

66. By Kepler's III[rd] law

$$\left(\frac{T_n}{T_s}\right)^2 = \left(\frac{R_n}{R_s}\right)^3$$

$$\frac{T_n}{T_5} = \left(\frac{R_n}{R_s}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$= 10\sqrt{10} = 10\times3.16 = 31.6$$

$$\therefore \qquad T_n : T_s = 36.6 : 1.$$

67. The magnitude of angular momentum at P is $L_p = m_p r_p v_p$

Similarly magnitude of angular momentum at A is $L_A = m_A r_A v_A$

From conservation of angular momentum

$$m_p r_p v_p = m_A v_A r_A$$

$$\frac{v_p}{v_A} = \frac{r_A}{r_p}$$

$\because \quad r_A > r_p, \therefore v_p > v_A$

area bound by SB and SC (SBAC > SBPC)

$\because$ By 2nd law equal areas are swept in equal intervals of time. Time taken to transverse BAC > time taken to transverse CPB.

❑❑