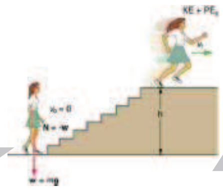


Unit - 4

# Work, Energy And Power

## 4.1 Introduction

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

## 4.2 Work Done by a Constant Force

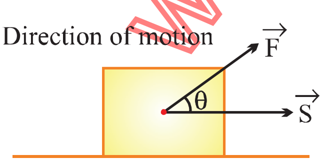
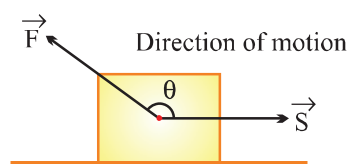
Let a constant force F be applied on the body such that it makes an angle $\theta$ with the horizontal and body is displaced through a distance *s*.

Then work done by the force in displacing the body through a distance *s* is given by

$$W = (F\cos\theta)\,s = Fs\cos\theta \Rightarrow W = (F\cos\theta)\,s = Fs\cos\theta$$

$$W = \vec{F}\cdot\vec{s}$$

## 4.3 Nature of Work Done

| Positive work | Negative work |
|---|---|
| Positive work means that force (or its component) is parallel to displacement $0^\circ \le \theta < 90^\circ$ | Negative work means that force (or its component) is opposite to displacement *i.e.*, $90^\circ < \theta \le 180^\circ$ |
|   |   |
| The positive work sigmnes that the external external force favours the motion of the body. | The negative work sigmnes that the force opposes the motion of the body. |

## 4.4 Work Done by aVariable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinite simal displacement is given by $dW = \vec{F}.\vec{ds}$.

The total work done in going from A to B is $W = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)\,ds$.

Area under force displacement curve with proper algebraic sign represents work done by the force.

## 4.5 Work Depends on Frame of Reference

With change of frame of reference (inertial) force does not change while displacement may change. So the work done by a force will be different in different frames.

*Examples :* If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ while in the frame of earth will be $\vec{F}.(\vec{s}+\vec{s_0})$ where $\vec{s_0}$ is the displacement of the train relative to the ground.

## 4.6 Energy

The energy of a body is defined as its capacity for doing work.

**(1)** It is a scalar quantity.

**(2)** Dimension : $[ML^2T^{-2}]$ it is same as that of work or torque.

**(3)** Units : Joule [S.I.], erg [C.G.S.]

Practical units : electron volt (eV), Kilowatt hour (KWh), Calories (Cal)

Relation between different units :

$$1 \text{ Joule} = 10^7 \text{ erg}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joule}$$

$$1 \text{ KWh} = 3.6 \times 10^6 \text{ Joule}$$

$$1 \text{ Calorie} = 4.18 \text{ Joule}$$

**(4)** Mass energy equivalence : The relation between the mass of a particle $m$ and its equivalent energy is given as $E = mc^2$ where $c$ = velocity of light in vacuum.

## 4.7 Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.

Let $m$ = mass of the body, $v$ = velocity of the body then K.E. = $\frac{1}{2}mv^2$.

**(1) Kinetic energy depends on frame of reference :** The kinetic energy of a person of mass $m$, sitting in a train moving with speed $v$, is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of the earth.

**(2) Work-energy theorem :** It states that work done by a force acting on a body is equal to the change produced in the kinetic energy of the body.

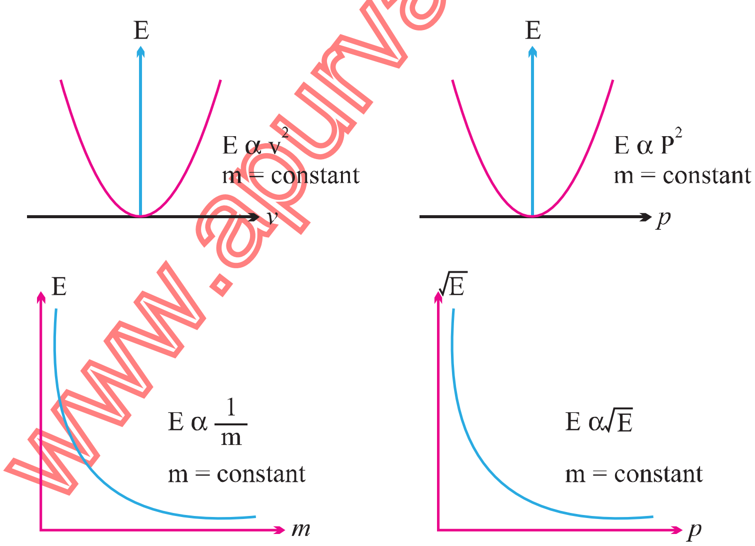
This theorem is valid for a system in presence of all types of forces (external or internal, conservative or non-conservative).

**(3) Relation of kinetic energy with linear momentum :** As we know

Momentum P = $\frac{2E}{V} = \sqrt{2mE}$

**(4) Various graphs of kinetic energy**

## 4.8 Potential Energy




## 4.9 Potential Energy

Potential energy is defined only for conservative forces. In the space occupied by conservative forces every point is associated with certain energy which is called the energy of position or potential energy. Potential energy generally are of three types : Elastic potential energy and Gravitational potential energy etc.

**(1) Change in potential energy :** Change in potential energy between any two points is defined in terms of the work done by the force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_1^2 \vec{F}.d\vec{r} = -W \qquad ...(1)$$

**(2) Potential energy curve :** A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve. Negative gradient of the potential energy gives force.

$$-\frac{dU}{dx} = F$$

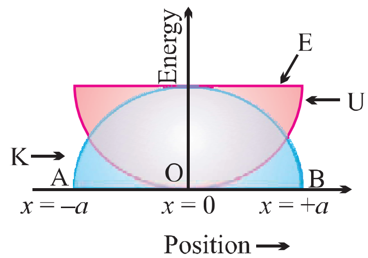
**(5) Types of equilibrium :** If net force acting on a particle is zero, it is said to be in equilibrium.

For equilibrium, $\frac{dU}{dx} = 0$, but the equilibrium of particle can be of three types :

| Stable | Unstable | Neutral |
|---|---|---|
| When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position. | When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium. | When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium. |
| Potential energy is minimum. | Potential energy is maximum. | Potential energy is constant. |
| $F = -\frac{dU}{dx} = 0$ | $F = -\frac{dU}{dx} = 0$ | $F = \frac{dU}{dx} = 0$ |
| $\frac{d^2U}{dx^2} = \text{positive}$ <br> *i.e.,* rate of change of $\frac{dU}{dx}$ is positive. | $\frac{d^2U}{dx^2} = \text{negative}$ <br> *i.e.,* rate of change of $\frac{dU}{dx}$ is negative. | $\frac{d^2U}{dx^2} = 0$ <br> *i.e.,* rate of change of $\frac{dU}{dx}$ is zero. |
| *Example :* A marble placed at the bottom of a hemispherical bowl. | *Example :* A marble balanced on top of a hemispherical bowl. | *Example :* A marble placed on horizontal table. |

## 4.10 Elastic Potential Energy

**(1) Restoring force and spring constant :** When a spring is stretched or compressed from its normal position ($x = 0$) by a small distance $x$, a restoring froce is produced in the spring to bring it to the normal position. According to Hooke's law this restoring force is proportional to the displacement $x$ and its direction is always opposite to the displacement.




*i.e.,* $\vec{F} \propto \vec{x}$

or $\vec{F} = k\vec{x}$ ...(i)

where $k$ is called spring constant.

**(2) Expression for elastic potential energy :**

Elastic potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2}Fx = \frac{F^2}{2k}$

**Note :**

- If spring is stretched from initial position $x_1$ to final position $x_2$ then work done = Increment in elastic potential energy

$$= \frac{1}{2}k(x_2^2 - x_1^2)$$

**(3) Energy graph for a spring :** It mean kinetic energy changes parabolically w.r.t. position but total energy remain always constant irrespective to position of the mass.

## 4.11 Law of Conservation of Energy

**(1) Law of conservation of energy :** For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

**(2) Law of conservation of total energy :** If the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical etc., which is conserved.

## 4.15 Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av}) = \dfrac{\Delta W}{\Delta t} = \dfrac{W}{t}$.

Instantaneous power $(P_{inst.}) = \dfrac{dW}{dt} = \dfrac{\vec{F}.d\vec{s}}{dt}$

[As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$.

*i.e.,* power is equal to the scalar product of force with velocity.

**(1)** Dimension : $[P] = [ML^2T^{-3}]$

**(2)** Units : Watt or Joule/sec [S.I.]

Practical Units : Kilowatt (kW), Mega watt (MW) and Horse power (*hp*)

Relations between different units : 1 watt = 1 Joule/sec = $10^7$ erg/sec

$1hp = 746$ Watt

**(3)** The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$

**(4)** Area under power time curve gives the work done as $P = \dfrac{dW}{dt}$

$\therefore \quad W = \int P dt$

$\therefore \quad W =$ Area under $P - t$ curve

## 4.12 Collision

Collision is an isolated event in which a strong force acts between two or more bodies for a short time as a result of which the energy and momentum of the interacting particle change.

In collision particles may or may not come in real touch.

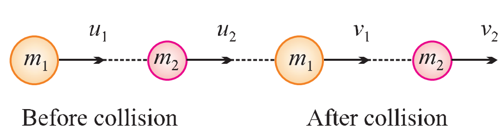
**(3) Types of collision :** (i) On the basis of conservation of kinetic energy.

| Perfectly Elastic collision | Inelastic collision | Perfectly inelastic collision |
|---|---|---|
| If in a collision, kinetic energy after collision is equal to kinetic energy before collision, the collision is said to be perfectly elastic. | If in a collision kinetic energy after collision is not equal to kinetic energy before collision, the collision is said to inelastic. | If in a collision two bodies stick together or move with same velocity after the collision, the collision is said to be perfectly inelastic. |
| Coefficient of restitution $e = 1$ | Coefficient of restitution $0 < e < 1$ | Coefficient of restitution $e = 0$ |
| $(KE)_{final} = (KE)_{initial}$ | Here kinetic energy appears in other forms. In some cases $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released. | The term 'perfectly inelastic' does not necessarily mean that all the initial kinetic energy is lost, it implies that the loss in kinetic energy is as large as it can be. (Consistent with momentum conservation). |
| **Examples : (1)** Collision between atomic particles<br>**(2)** Bouncing of ball with same velocity after the collision with earth. | **Examples :** (1) Collision between two billiard balls.<br>(2) Collision between two automobile on a road.<br>In fact all majority of collision belong to this category. | **Example :** Collision between a bullet and a block of wood into which it is fired. When the bullet remains embeded in the block. |



## 4.13 Perfectly Elastic Head on Collision

Let two bodies of masses $m_1$ and $m_2$ moving initial velocities $u_1$ and $u_2$ in the same direction they collide such that after collision their final velocities are $v_1$ and $v_2$ respectively.



According to law of conservation of momentum and conservation of kinetic energy.

**Note :**

- The ratio of relative velocity of separation and relative velocity of approach is defined as coefficient of restitution.

$$e = \frac{v_2 - v_1}{u_1 - u_2} \text{ or } v_2 - v_1 = e\,(u_1 - u_2).$$

- For perfectly elastic collision $e = 1$

$\therefore \quad v_2 - v_1 = u_1 - u_2$ [As shown in eq. (vi)]

- For perfectly inelastic collision $e = 0$

$\therefore \quad v_2 - v_1 = 0$ or $v_2 = v_1$

It means that two body stick together and move with same velocity.

- For inelastic collision $0 < e < 1$

$\therefore \quad v_2 - v_1 = (u_1 - u_2)$

In short we can say that $e$ is the degree of elasticity of collision and it is dimension less quantity.

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2} \qquad \text{...(vii)}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_1 + \frac{2m_1u_1}{m_1 + m_2} \qquad \text{...(viii)}$$

- When two bodies of equal masses undergo head on elastic collision, their velocities get interchanged.

**(2) Kinetic energy transfer during head on elastic collision :** Fractional decrease in kinetic energy

$$\frac{\Delta K}{K} = \frac{4m_1m_2}{(m_1 - m_2) + 4m_1m_2} \qquad \text{...(iv)}$$

**Note :**

- Greater the difference in masses less will be transfer of kinetic energy and vice versa.

- Transfer of kinetic energy in head on elastic collision (when target is at rest) is maximum when the masses of particles are equal.

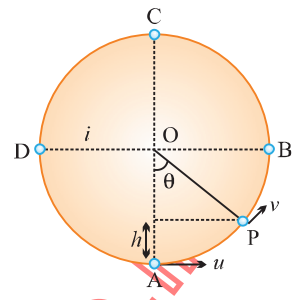
## 2.14 Motion in Vertical Circle

This is an example of non-uniform circular motion. In this motion body is under the influence of gravity of earth.

(1) **Velocity at any point on vertical loop :** If $u$ is the initial velocity imparted to body at lowest point then, velocity of body at height $h$ is given by

$$v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2gl(1-\cos\theta)}$$

where $l$ is the length of the string.




(2) **Tension at any point on vertical loop :** Tension at general point P,

$$T = mg\cos\theta + \frac{mv^2}{l}$$

(3) **Various conditions for vertical motion :**

| Velocity at lowest point | Condition |
|---|---|
| $u_A > \sqrt{5gl}$ | Tension in the string will not be zero at any of the point and body will continue the circular motion. |
| $u_A = \sqrt{5gl}$, | Tension at highest point C will be zero and body will just complete the circle. |
| $\sqrt{2gl} < u_A < \sqrt{5gl}$, | Particle will not follow circular motion. Tension in string become zero somewhere between points B and C whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory. |
| $u_A = \sqrt{2gl}$ | Both velocity and tension in the string becomes zero between A and B and particle will oscillate along semi-circular path. |
| $u_A < \sqrt{2gl}$ | Velocity of particle becomes zero between A and B but tension will not be zero and the particle will oscillate about the point A. |

**(6) Various quantities for a critical condition in a vertical loop at different positions :**

| Quantity | Point A | Point B | Point C | Point D | Point P |
|---|---|---|---|---|---|
| Linear velocity ($v$) | $\sqrt{5gl}$ | $\sqrt{3gl}$ | $\sqrt{gl}$ | $\sqrt{3gl}$ | $\sqrt{gl(3+2\cos\theta)}$ |
| Angular velocity ($\omega$) | $\sqrt{\frac{5g}{l}}$ | $\sqrt{\frac{3g}{l}}$ | $\sqrt{\frac{g}{l}}$ | $\sqrt{\frac{3g}{l}}$ | $\sqrt{\frac{g}{l}(3+2\cos\theta)}$ |
| Tension in String (T) | $6\,mg$ | $3\,mg$ | 0 | $3\,mg$ | $3\,mg(1+\cos\theta)$ |
| Kinetic Energy (KE) | $\frac{5}{2}mgl$ | $\frac{3}{2}mgl$ | $\frac{1}{2}mgl$ | $\frac{3}{2}mgl$ | $\frac{mu^2}{1} - 5mg = 0$ |
| Potential Energy (PE) | 0 | $mgl$ | $2mgl$ | $mgl$ | $mgl(1-\cos\theta)$ |
| Total Energy (TE) | $\frac{5}{2}mgl$ | $\frac{5}{2}mgl$ | $\frac{5}{2}mgl$ | $\frac{5}{2}mgl$ | $\frac{5}{2}mgl$ |

## VERY SHORT ANSWER QUESTIONS (1 MARK)

1. Define the conservative and non-conservative forces. Give examples of each.
2. A light body and a heavy body have same linear momentum. Which one has greater K.E ?
3. The momentum of the body is doubled, what % does its K.E change ?
4. A body is moving along a circular path. How much work is done by the centripetal force ?
5. Which spring has greater value of spring constant – a hard spring or a delicate spring ?
6. Two bodies stick together after collision. What type of collision is in between these two bodies ?
7. State the two conditions under which a force does no work ?
8. How will the momentum of a body changes if its K.E. is doubled ?
9. K.E. of a body is increased by 300 %. Find the % increase in its momentum ?
10. A light and a heavy body have same K.E., which of the two have more momentum and why ?

**11.** Does the P.E. of a spring decreases or increases when it is compressed or stretched ?

**12.** Name a process in which momentum changes but K.E. does not.

**13.** What happens to the P.E. of a bubble when it rises in water ?

**14.** A body is moving at constant speed over a frictionless surface. What is the work done by the weight of the body ?

**15.** Define spring constant of a spring.

## SHORT ANSWER QUESTIONS (2 MARKS)

**16.** How much work is done by a coolie walking on a horizontal platform with a load on his head ? Explain.

**17.** Mountain roads rarely go straight up the slope, but wind up gradually. Why ?

**18.** A truck and a car moving with the same K.E. on a straight road. Their engines are simultaneously switched off which one will stop at a lesser distance ?

**19.** Is it necessary that work done in the motion of a body over a closed loop is zero for every force in nature ? Why?

**20.** Derive an expression for K.E. of a body of mass '$m$' moving with velocity '$v$' by calculus method.

**21.** How high must a body be lifted to gain an amount of P.E. equal to the K.E. it has when moving at speed 20 $ms^{-1}$. (The value of acceleration due to gravity at a place is 9.8 $ms^{-2}$).

**22.** Give an example in which a force does work on a body but fails to change its K.E.

**23.** A bob is pulled sideway so that string becomes parallel to horizontal and released. Length of the pendulum is 2 m. If due to air resistance loss of energy is 10%, what is the speed with which the bob arrived at the lowest point.

**24.** Two springs A and B are identical except that A is harder than B ($K_A > K_B$) if these are stretched by the equal force. In which spring will more work be done ?

**25.** Find the work done if a particle moves from position $r_1 = (3\hat{i}+2\hat{j}-6\hat{k})$ to a position $\vec{r}_2 = (14\hat{i}+13\hat{j}-9\hat{k})$ under the effect of force $\vec{F} = (4\hat{i}+\hat{j}+3\hat{k})$N.

**26.** Spring A and B are identical except that A is stiffer than B, *i.e.,* force constant $k_A > k_B$. In which spring is more work expended if they are stretched by the same amount ?

**27.** A ball at rest is dropped from a height of 12 m. It loses 25% of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy ?

**28.** State and prove work energy theorem.

**29.** Which of the two kilowatt hour or electron volt is a bigger unit of energy and by what factor ?

**30.** A spring of force constant K is cut into two equal pieces. Calculate force constant of each part.

## SHORT ANSWER QUESTIONS (3 MARKS)

**31.** A elastic spring is compressed by an amount $x$. Show that its P.E. is $1/2\ kx^2$ where $k$ is the spring constant.

**32.** A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel ? What is the power supplied by each crane ? Neglect Power dissipation against friction.

**33.** Prove that bodies of identical masses exchange their velocities after head-on elastic collision.

**34.** Answer the following :

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere or both ?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why ?

**35.** Define elastic and inelastic collision. A lighter body collides with a much more massive body at rest. Prove that the direction of lighter body is reversed and massive body remains at rest.

**36.** 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is further stretched through 0.1 m. Calculate work done.

**37.** A body of mass M at rest is struck by a moving body of mass m. Prove that fraction of the initial K.E. of the mass m transferred to the struck body is $4\ m\ M/(m + M)^2$ in an elastic collision.

**38.** A pump on the ground floor of a building can pump up water to fill a tank of volume 30 $m^3$ in 15 min. If the tank is 40 m above the ground, how much electric power is consumed by the pump. The efficiency of the pump is 30%.

**39.** Show that in an elastic one dimensional collision the relative velocity of approach before collision is equal to the relative velocity of separation after collision.

**40.** A ball bounces to 80% of its original height. Calculate the mechanical energy lost in each bounce.

## LONG ANSWER QUESTIONS (5 MARKS)

**41.** Show that at any instant of time during the motion total mechanical energy of a freely falling body remains constant. Show graphically the variation of K.E. and P.E. during the motion.

**42.** Define spring constant, write the characteristics of the force during the elongation of a spring. Derive the relation for the P.E. stored when it is elongated by X. Draw the graphs to show the variation of P.E. and force with elongation.

**43.** How does a perfectly inelastic collision differ from perfectly elastic collision ? Two particles of mass $m_1$ and $m_2$ having velocities $U_1$ and $U_2$ respectively make a head on collision. Derive the relation for their final velocities. Discuss the following special cases.

(i) $m_1 = m_2$

(ii) $m_1 >> m_2$ and $U_2 = 0$

(iii) $m_1 << m_2$ and $U_1 = 0$

## NUMERICALS

**44.** A body is moving along $z$-axis of a coordinate system under the effect of a constant force $F = (2\hat{i} + 3\hat{j} + \hat{k})$N. Find the work done by the force in moving the body a distance of 2 m along $z$-axis.

**45.** Water is pumped out of a well 10 m deep by means of a pump rated 10 KW. Find the efficiency of the motor if 4200 kg of water is pumped out every minute. Take $g = 10$ m/$s^2$.

**46.** A railway carriage of mass 9000 kg moving with a speed of 36 km $h^{-1}$ collides with a stationary carriage of same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision ? What type of collision is this ?

**47.** In lifting a 10 kg weight to a height of 2m, 230 J energy is spent. Calculate the acceleration with which it was raised ?

**48.** A bullet of mass 0.02 kg is moving with a speed of 10 $ms^{-1}$. It can penetrate 10 cm of a wooden block, and comes to rest. If the thickness of the target would be 6 cm only, find the K.E. of the bullet when it comes out.

**49.** A man pulls a lawn roller through a distance of 20 m with a force of 20 kg weight. If he applies the force at an angle of 60º with the ground, calculate the power developed if he takes 1 min in doing so.

**50.** A body of mass 0.3 kg is taken up an inclined plane to length 10 m and height 5 m and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the

(i) work done by the gravitational force over the round trip.

(ii) work done by the applied force over the upward journey.

(iii) work done by frictional force over the round trip.

(iv) kinetic energy of the body at the end of the trip.

How is the answer to (iv) related to the first three answer ?

**51.** Two identical 5 kg blocks are moving with same speed of 2 $ms^{-1}$ towards each other along a frictionless horizontal surface. The two blocks collide, stick together and come to rest. Consider the two blocks as a system. Calculate work done by (i) external forces and (i) Internal forces.

**52.** A truck of mass 1000 kg accelerates uniformly from rest to a velocity of 15 $ms^{-1}$ in 5 seconds. Calculate (i) its acceleration, (ii) its gain in K.E., (iii) average power of the engine during this period, neglect friction.

**53.** An elevator which can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2 $ms^{-1}$. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power.

**54.** To simulate car accidents, auto manufacturers study the collisions of moving cars with mounted springs of different spring constants. Consider a typical simulation with a car of mass 1000 kg moving with a speed 18.0 $kmh^{-1}$ on a smooth road and colliding with a horizontally mounted spring of spring constant $6.25 \times 10^{-3}$ $Nm^{-1}$. What is the maximum compression of the spring.

## ANSWERS

1. **Conservative force :** *e.g.,* Gravitational force, electrostatic force.

   **Non-Conservative force :** *e.g.,* forces of friction, viscosity.

2. Lighter body has more K.E. as K.E. $= \dfrac{p^2}{2m}$ and for constant $p$, K.E. $\propto \dfrac{1}{m}$.

3. K.E. $= \dfrac{p^2}{2m}$ when $p$ is doubled K.E. becomes 4 times.

   $$\therefore \quad \text{\% Increase in K.E.} = \frac{\Delta \text{K.E.}}{\text{K.E.}} \times 100 = \frac{4\text{K.E.} - \text{K.E.}}{\text{K.E.}} \times 100$$

   $$= 3 \times 100 = 300\%.$$

4. $W = FS \cos 90^\circ = 0$.

5. Hard spring.

6. Inelastic collision.

7. (i) Displacement is zero or it is perpendicular to force.

   (ii) Conservative force moves a body over a closed path.

8. Momentum becomes $\sqrt{2}$ times.

9. K.E. $= \dfrac{p^2}{2m}$ so $p = \sqrt{2mk}$

   $$\text{Increase in K.E.} = 300\% \text{ of } k = 3k$$

   $$\text{Final K.E., } k' = k + 3k = 4k$$

   $$\text{Final momentum, } p' = \sqrt{2mk'} = \sqrt{2m \times 4k} = 2\sqrt{2mk}$$

   $$= 2p$$

   $$\text{\% Increase in momentum} = \frac{p' - p}{p} \times 100 = 100\%$$

10. Heavier body.

11. Increases because W.D. on it when it is compressed or stretched.

12. Uniform circular motion.

13. Decreases.

**14.** $W = 0$.

**15.** It is the restoring force set up in a string per unit extension.

**16.** $W = 0$ as his displacement is along the horizontal direction and in order to balance the load on his head, he applies a force on it in the upward direction equal to its weight. Thus angle between force and displacement is zero.

**17.** If roads go straight up then angle of slope $\theta$ would be large so frictional force $f = \mu mg \cos\theta$ would be less and the vehicles may slip. Also greater power would be required.

**18.** By Work - Energy Theorem,

Loss in K.E. = W.D. against the force × distance of friction

or $\quad \text{K.E.} = \mu mg S$

For constant K.E., $\quad S \propto \dfrac{1}{m}$

$\therefore$ Truck will stop in a lesser distance.

**19.** No. W.D. is zero only in case of a conservative force.

**21.** $$mgh = \frac{1}{2}mv^2$$

so $\quad h = 20.2$ m

**22.** When a body is pulled on a rough, horizontal surface with constant velocity. Work is done on the body but K.E. remains unchanged.

**23.** $$\frac{1}{2}mv^2 = 90\% \text{ of } mgh$$

$\therefore \quad v = 6$ m/s

**24.** $F = Kx$ so $x = \dfrac{F}{K}$

For same F, $$W_A = \frac{1}{2}K_A x^2 = \frac{1}{2}\frac{F^2}{K_A}$$

and $$W_B = \frac{F^2}{2K_B}$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{K_B}{K_A}$$

As $K_A > K_B$ so $W_A < W_B$.

**25.** $$\vec{r} = \vec{r}_2 - \vec{r}_1 = 11\hat{i} + 11\hat{j} - 3\hat{k}$$

$$\vec{F} = (4\hat{i}+\hat{j}+3\hat{k})\text{N}$$

$$\therefore \quad W = \vec{F}.\vec{r} = 46 \text{ J.}$$

**26.** $$W = \frac{1}{2}Kx^2$$

$$\therefore \quad \frac{W_A}{W_B} = \frac{K_A}{K_B}, \text{ for same } x$$

As $K_A > K_B$ so $W_A > W_B$.

**27.** If ball bounces to height $h'$, then

$$mgh' = 75\% \text{ of } mgh$$

$$\therefore \quad h' = 0{\cdot}75\, h = 9 \text{ m.}$$

**29.** *kwh* is a bigger unit of energy.

$$\frac{1kwh}{1eV} = \frac{3{\cdot}6\times10^6 \text{ J}}{1{\cdot}6\times10^{-19} \text{ J}} = 2{\cdot}25\times10^{25}$$

**30.** Force constant of each half becomes twice the force constant of the original spring.

**32.** $t_1 = 1$ min $= 60$ s, $t_2 = 2$ min $= 120$ s

$W = F_s = mgs = 5{\cdot}88 \times 10^5$ J

As both cranes do same amount of work so both consume same amount of fuel.

$$P_1 = \frac{W}{t_1} \text{ and } P_1 = \frac{W}{t_2}$$

$$\therefore \quad P_1 = 9800 \text{ W } \& \text{ } P_2 = 4900 \text{ W.}$$

**36.** P.E. of spring when stretched through a distance $0{\cdot}1$ m,

$$U = \text{W.D.} = \frac{1}{2}Kx^2 = 20 \text{ J}$$

or $$K = 4000 \text{ N/m}$$

when spring is further stretched through $0{\cdot}1$ m, then P.E. will be :

$$U' = \frac{1}{2}k(0{\cdot}2)^2 = 80 \text{ J}$$

$$\therefore \quad \text{W.D.} = U' - U = 80 - 20 = 60 \text{ J.}$$

**38.** $$30\% \text{ of Power} = \frac{W}{t} = \frac{mgh}{t} = \frac{V\rho gh}{t}$$

$$\frac{30}{100} \times P = \frac{V\rho gh}{t}$$

$$\therefore \quad P = 43{\cdot}6 \text{ KW.}$$

**40.** Let Initial P.E. = $mgh$

$$\text{P.E. after first bounce} = mg \times 80\% \text{ of } h$$

$$= 0{\cdot}80\, mgh$$

$$\text{P.E. lost in each bounce} = 0{\cdot}20\, mgh$$

$\therefore$ Fraction of P.E. lost in each bounce

$$= \frac{0{\cdot}20mgh}{mgh} = 0{\cdot}20$$

**44.** $$\vec{F} = (2\hat{i} + 3\hat{j} + \hat{k})\text{N}, \ \vec{S} = 2\hat{k}$$

$$W = \vec{F}.\vec{S} = 2 \text{ J.}$$

**45.** $$\text{Input power} = 10 \text{ KW}$$

$$\text{Output power} = \frac{W}{t} = \frac{mgh}{t} = 7 \text{ KW}$$

$$\therefore \quad \text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = 70\%$$

**46.** $m_1 = 9000$ kg, $u_1 = 36$ km/$h$ = 10 m/s

$m_2 = 9000$ kg, $u_2 = 0$, $v = v_1 = v_2 = ?$

By conservation of momentum :

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$$

$$\therefore \quad v = 5 \text{ m/s}$$

$$\text{Total K.E. before collision} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= 45000 \text{ J}$$

$$\text{Total K.E. after collision} = \frac{1}{2}(m_1 + m_2)v^2 = 225000 \text{ J}$$

As total K.E. after collision < Total K.E. before collision

$\therefore$ Collision is inelastic.

**47.** $$W = mgh + mah = m(g + a)h$$

$\therefore$ $$a = 1{\cdot}5 \text{ m/s}^2.$$

**48.** For $x = 10$ cm $= 0{\cdot}1$ m, $Fx = \frac{1}{2}mv_1^2 = 1$ J

$\therefore$ $$F = 10 \text{ N}$$

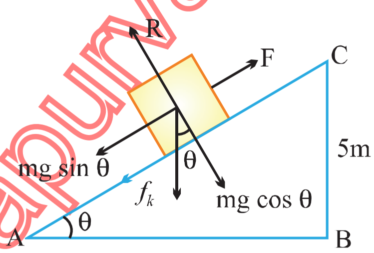
For $x = 6$ cm $= 0{\cdot}06$ m, $Fx = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$

or $$Fx = \frac{1}{2}mv_1^2 - \text{Final K.E.}$$

or $$\text{Final K.E.} = \frac{1}{2}mv_1^2 - Fx = 1 - 10 \times 0{\cdot}06$$
$$= 1 - 0{\cdot}6$$
$$= 0{\cdot}4 \text{ J}$$

**49.** $P = \frac{W}{t} = \frac{Fs\cos\theta}{t} = 32{\cdot}66$ W

**50.**




$$\sin\theta = \frac{CB}{CA} = 0{\cdot}5$$

$\therefore$ $$\theta = 30^\circ.$$

(i) W = FS = – $mg$ sin q × $h$ = – 14·7 J is the W.D. by gravitational force in moving the body up the inclined plane.

W′ = FS = + $mg$ sin θ × $h$ = 14·7 J is the W.D. by gravitational force in moving the body down the inclined plane.

$\therefore$ Total W.D. round the trip, $W_1 = W + W' = 0$.

(ii) Force needed to move the body up the inclined plane,

$$F = mg\sin\theta + f_k$$
$$= mg\sin\theta + \mu_k R$$
$$= mg\sin\theta + \mu_k mg\cos\theta$$

$\therefore$ W.D. by force over the upward journey is

$$W_2 = F \times l = mg(\sin\theta + \mu_k\cos\theta)\,l$$
$$= 18{\cdot}5\text{ J}$$

(iii) W.D. by frictional force over the round trip,

$$W_3 = -f_k(l + l) = -2f_k l$$
$$= -2\mu_k mg\cos\theta\, l = -7{\cdot}6\text{ J}$$

(iv) K.E. of the body at the end of round trip

= W.D. by net force in moving the body down the inclined plane

$$= (mg\sin\theta - \mu_k mg\cos\theta)\,l$$
$$= 10{\cdot}9\text{ J}$$

$\Rightarrow$ K.E. of body = net W.D. on the body.

**51.** Here no external forces are acting on the system so :

$$\vec{F}_{ext.} = 0 \Rightarrow W_{ext.} = 0$$

According to work-energy theorem :

Total W.D. = Change in K.E.

or $W_{ext.} + W_{int.}$ = Final K.E. – Initial K.E.

$$0 + W_{int.} = 0 - \left(\frac{1}{2}mu^2 + \frac{1}{2}mu^2\right)$$

or $W_{int.} = -mu^2 = -20$ J

**52.** (i) $a = \dfrac{v-u}{t} = 3\text{ m/s}^2$

(ii) Gain in K.E. $= \dfrac{1}{2}m(v^2 - u^2) = 1{\cdot}125 \times 10^5$ J

(iii) $P = \frac{W}{t} = 22500$ W

**53.** Downward force on the elevator is :

$$F = mg + f = 22000 \text{ N}$$

$\therefore$ Power supplied by motor to balance this force is :

$$P = Fv = 44000 \text{ W}$$

$$= \frac{44000}{746} = 59\ hp.$$

**54.** At maximum compression $x_m$, the K.E. of the car is converted entirely into the P.E. of the spring.

$\therefore$ $$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2$$

or $$x_m = 2 \text{ m.}$$

❑❑