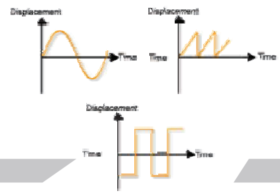



Unit - 10

# Oscillations
# Waves

## 10.1 Periodic Motion

A motion, which repeat itself over and over again after a regular interval of time is called a periodic motion and the fixed interval of time after which the motion is repeated is called period of the motion. *Examples :* Revolution of earth around the sun (period one year).

## 10.2 Oscillatory or Vibratory Motion.

The motion in which a body moves to and fro or back and forth repeatedly about a fixed point in a definite interval of time. Oscillatory motion is also called as harmonic motion. *Example :* The motion of the pendulum of a wall clock.

## 10.3 Harmonic and Non-harmonic Oscillation.

Harmonic oscillation is that oscillation which can be expressed in terms of single harmonic function (*i.e.* sine or cosine function). *Example :* $y = a \sin \omega t$ or $y = a \cos \omega t$.

Non-harmonic oscillation is that oscillation which can not be expressed in terms of single harmonic function. *Example :* $y = a \sin \omega t + b \sin 2\omega t$.

## 10.4 Some Important Definitions.

(1) **Time period :** It is the least interval of time after which the periodic motion of a body repeats itself. S.l. units of time period is second.

(2) **Frequency :** It is defined as the number of periodic motions executed by body per second. S.l unit of frequency is hertz (Hz).

(3) **Angular Frequency :** $\omega = 2\varpi n$

(4) **Displacement:** Its deviation from the mean position.

**(5) Phase :** It is a physical quantity, which completely express the position and direction of motion, of the particle at that instant with respect to its mean position.

$Y = a \sin \theta = a \sin (\omega t + \phi_0)$ here $\theta = \omega t + \phi_0$ = phase of vibrating particle.

**(i) Initial phase or epoch :** It is the phase of a vibrating particle at $t = 0$.

**(ii) Same phase:** Two vibrating particle are said to be in same phase, if the phase difference between them is an even multiple of $n$ or path difference is an even multiple of $(\lambda/2)$ or time interval is an even multiple of (T/2).

**(iii) Opposite phase :** Opposite phase means the phase difference between the particle is an odd multiple of $\varpi$ or the path difference is an odd multiple of $\lambda$ or the time interval is an odd multiple of (T/2).

**(iv) Phase difference :** If two particles performs S.H.M and their equation are $y_1 = a \sin (\omega t + \phi_1)$ and $y_2 = a \sin (\omega t + \phi_2)$ then phase difference $\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$

## 10.5 Simple Harmonic Motion.

Simple harmonic motion is a special type of periodic motion, in which Restoring force $\propto$ Displacement of the particle from mean position.

$$F = -kx$$

Where $k$ is known as force constant. Its S.l. unit is Newton/meter and dimension is $[MT^{-2}]$.

## 10.6 Displacement in S.H.M.

Simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference

(i) $y = a \sin \omega t$ when at $t = 0$ the vibrating particle is at mean position.

(ii) $y = a \cos \omega t$ when at $t = 0$ the vibrating particle is at extreme position.

(iii) $y = a \sin (\omega t \pm \phi)$ when the vibrating particle is $\phi$ phase leading or lagging from the mean position.

## 10.7 Comparative Study of Displacement, Velocity and Acceleration.

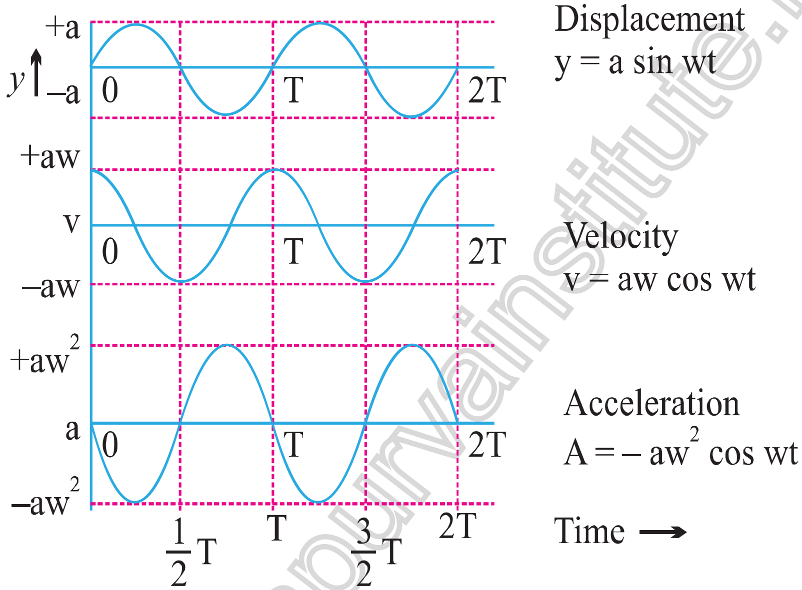
Displacement $y = a \sin \omega t$

Velocity $v = a\omega \cos \omega t$

$$\omega t = a\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

Acceleration $A = -a\omega^2 \sin \omega t$

$$\omega t = a\omega^2 \sin(\omega t + \pi)$$




(i) All the three quantities displacement, velocity and acceleration show harmonic variation with time having same period.

(ii) The velocity amplitude is $\omega$ times the displacement amplitude

(iii) The acceleration amplitude is $\omega^2$ times the displacement amplitude

(iv) In S.H.M. the velocity is ahead of displacement by a phase angle $\pi/2$.

(v) In S.H.M. the acceleration is ahead of velocity by a phase angle $\pi/2$.

(vi) The acceleration is ahead of displacement by a phase angle of $\pi$.

(vii) Various physical quantities in S.H.M. at different position :

| Physical quantities | Equilibrium position ($y = 0$) | Extreme Position ($y = \pm a$) |
|---|---|---|
| Displacement $y = a \sin \omega t$ | Minimum (Zero) | Maximum ($a$) |
| Velocity $v = \omega\sqrt{a^2 - y^2}$ | Maximum ($a\omega$) | Minimum (Zero) |
| Acceleration $A = -\omega^2 y$ | Minimum (Zero) | Maximum ($\omega^2 a$) |

## 10.8 Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy : Potential energy and Kinetic energy

**(1) Potential energy :** $U = \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t$

(i) $U_{max} = \frac{1}{2} ka^2 = \frac{1}{2} m\omega^2 a^2$ when $y = \pm a$; $\omega t = \pi/2$; $t = T/4$

(ii) $U_{min} = 0$ when $y = 0$; $\omega t = 0$; $t = 0$

**(2) Kinetic energy :**

$K = \frac{1}{2} ma^2\omega^2 \cos^2 \omega t$ or $K = \frac{1}{2} m\omega^2 (a^2 - y^2)$

(i) $K_{max} = \frac{1}{2} m\omega^2 a^2$ when $y = 0$; $t = 0$; $\omega t = 0$

(ii) $K_{min} = 0$ when $y = a$; $t = T/4$, $\omega t = \pi/2$

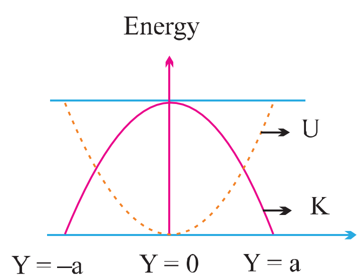
**(3) Total energy :** Total mechanical energy

= Kinetic energy + Potential energy

$$E = \frac{1}{2} m\omega^2 a^2$$

Total energy is not a position function *i.e.* it always remains constant.

**(4) Energy position graph :**



**(5)** Kinetic energy and potential energy vary periodically double the frequency of S.H.M.

## 10.9 Time Period and Frequency of S.H.M.

Time period (T) = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ as $\omega\sqrt{\frac{k}{m}}$

Frequency ($n$) = $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

In general $m$ is called inertia factor and $k$ is called spring factor.

Thus $T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

## 10.10 Differential Equation of S.H.M.

For S.H.M. (linear) $m\frac{d^2y}{dt^2} + ky = 0$ [As $\omega = \sqrt{\frac{k}{m}}$]

For angular S.H.M. $\frac{d^2\theta}{dt^2} + \omega^2\theta = 0$ [$\omega^2 = \frac{c}{l}$]

## 10.11 Simple Pendulum

Mass of the bob = $m$

Effective length of simple pendulum = 1; $T = 2\pi\sqrt{\frac{l}{g}}$

(i) Time period of simple pendulum is independent of amplitude as long as its motion is simple harmonic.

(ii) Time period of simple pendulum is also independent of mass of the bob.

(iii) If the length of the pendulum is comparable to the radius of earth then $T = 2\varpi\sqrt{\frac{1}{g\left[\frac{1}{l}+\frac{1}{R}\right]}}$

If $l >> R$ $(\rightarrow\infty)$ $1/l < 1/R$ so $T = 2\varpi\sqrt{\frac{R}{g}} \cong$ 84.6 minutes

(iv) The time period of simple pendulum whose point of suspension moving horizontally with acceleration,

$$a\ \mathrm{T} = 2\varpi\sqrt{\frac{l}{(g^2 + a^2)^{1/2}}} \quad \text{and} \quad \theta = \tan^{-1}(a/g)$$

(v) Second's Pendulum : It is that simple pendulum whose time period of vibrations is two seconds.

(vi) Work done in giving an angular displacement $\theta$ to the pendulum from its mean position.

$$\mathrm{W} = \mathrm{U} = mgl\,(1 - \cos\theta)$$

(vii) Kinetic energy of the bob at mean position = work done or potential energy at extreme

(viii) Various graph for simple pendulum.

## 10.12 Spring Pendulum

A point mass suspended from a mass less spring or placed on a frictionless horizontal plane attached with spring constitutes a linear harmonic spring pendulum

$$\text{Time period } T = 2\varpi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$T = 2\varpi\sqrt{\frac{m}{k}} \quad \text{and Frequency } n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

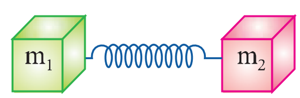
(i) Time of a spring pendulum is independent of acceleration due to gravity.

(ii) If the spring has a mass $M$ and mass $m$ is suspended from it, effective

mass is given by $m_{eff} = m + \frac{M}{3}$

So that $T = 2\pi\sqrt{\frac{m_{eff}}{k}}$

(iii) If two masses of mass $m_1$ and $m_2$ are connected by a spring and made to oscillate on horizontal surface, the reduced mass $m_r$ is given by




$$\frac{1}{m_r} = \frac{1}{m_1} + \frac{1}{m_2}$$

So that $T = 2\pi\sqrt{\frac{m_r}{k}}$

(iv) If a spring pendulum, oscillating in a vertical plane is made to oscillate on a horizontal surface, (or on inclined plane) time period will remain unchanged.

(v) If the stretch in a vertically loaded spring is $y_0$ then

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{y_0}{g}}$$

Time period does not depends on '$g$' because along with g, $y_0$ will also change in such a way that $\frac{y_0}{g} = \frac{m}{k}$ remains constant.

(vi) Series combination : If $n$ springs of different force constant are connected in series having force constant $k_1$, $k_2$, $k_3$ ........ respectively

then $$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ......$$

(vii) Parallel combination: If the springs are connected in parallel then $k_{eff} = k_1 + k_2 + k_3 + ........$

(viii) If the spring of force constamt $k$ is divided in to $n$ equal parts then spring constant of each part will become $nk$.

(ix) The spring constant $k$ is inversely proportional to the spring length.

As $k \; \alpha \; \frac{1}{\text{Extension}} \; \alpha \; \frac{1}{\text{Length of spring}}$

(x) When a spring of length $l$ is cut in two pieces of length $l_1$ and $l_2$ such that $l_1 = nl_2$.

If the constant of a spring is $k$ then ppring constant of first part $k_1 = \frac{k(n+1)}{n}$

Spring constant of second part $k_2 = (n + 1)\, k$ and ratio of spring constant $\frac{k_1}{k_2} = \frac{1}{n}$.

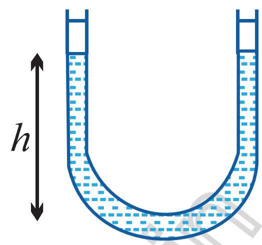
## 10.13 Various Formulae of S.H.M. .

**S.H.M. of a liquid in U tube :**

If a liquid of density ρ contained in a vertical U tube performs S.H.M. in its two limbs. Then time period

$$T = 2\pi\sqrt{\frac{L}{2g}} = 2\pi\sqrt{\frac{h}{g}}$$

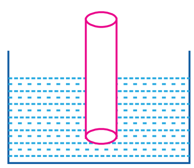
Where $L$ = Total length of liquid column, $H$ = Height of undisturbed liquid in each limb ($L = 2h$)




**S.H.M. of a floating cylinder**

If $l$ is the length of cylinder dipping in liquid then time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

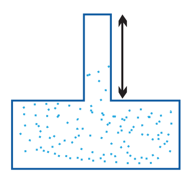


**S.H.M. of ball in the neck of an air chamber**

**Image**

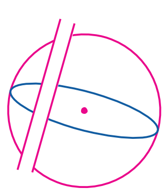
$$T = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$$

$M$ = mass of the ball
$V$ = volume of air chamber
$A$ = area of cross section of neck
$E$ = Bulk modulus for Air



S.H.M. of a body in a tunnel dug along any chord of earth

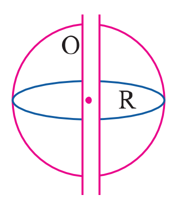
$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ minutes}$$



S.H.M. of body in the tunnel dug along the diameter of earth

$$T = 2\pi\sqrt{\frac{R}{g}}$$

$T$ = 84.6 minutes
$R$ = radius of the earth = 6400 km
$g$ = acceleration due to gravity = 9.8 m/s$^2$ at earth's surface



## 10.14 Free, Damped, Forced and Maintained Oscillation.

**(1) Free oscillation**

(i) The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations

(ii) The amplitude, frequency and energy of oscillation remains constant

(iii) Frequency of free oscillation is called natural frequency.

**(2) Damped oscillation**

(i) The oscillation of a body whose amplitude goes on decreasing with time are defined as damped oscillation.

(ii) Amplitude of oscillation decreases exponentially due to damping forces like frictional force, viscous force, hystersis etc.

**(3) Forced oscillation**

(i) The oscillation in which a body oscillates under the influence of an external periodic force are known as forced oscillation.

(ii) Resonance: When the frequency of external force is equal to the natural frequency of the oscillator. Then this state is known as the state of resonance. And this frequency is known as resonant frequency.

**(4) Maintained oscillation :** The oscillation in which the loss of oscillator is compensated by the supplying energy from an external source are known as maintained oscillation.

## 10.15 Wave

A wave is a disturbance which propagates energy and momentum from one place to the other without the transport of matter.

**(1) Necessary properties of the medium for wave propagation :**

(i) *Elasticity :* So that particles can return to their mean position, after having been disturbed.

(ii) *Inertia :* So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

**(2) Mechanical waves :** The waves which require medium for their propagation are called mechanical waves.

*Example :* Waves on string and spring, waves on water surface, sound waves, seismic waves.

**(3) Non-mechanical waves :** The waves which do not require medium for their propagation are called non-mechanical or electromagnetic waves.

*Examples :* Light, heat (Infrared), radio waves, $\gamma$-rays. X-rays *etc.*

**(4) Transverse waves :** Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

(ii) A crest is a portion of the medium which is raised temporarily.

(iii) A trough is a portion of the medium which is depressed temporarily.

(iv) Examples of transverse wave motion: Movement of string of a sitar, waves on the surface of water.

(v) Transverse waves can not be transmitted into liquids and gases.

**(5) Longitudinal waves:** If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.

(i) It travels in the form of compression and rarefaction.

(ii) A compression © is a region of the medium in which particles are compressed.

(iii) A rarefaction (R) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves.

(v) These waves can be transmitted through solids, liquids and gases.

## 10.16 Important Terms

**(1) Wavelength :**

(i) It is the length of one wave.

(ii) Distance travelled by the wave in one time period is known as wavelength.

$\lambda$ = Distance between two consecutive crests or troughs.

**(2) Frequency :** Number of vibrations completed in one second.

**(3) Time period :** Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

**(4) Relation between frequency and time period :**

Time period = 1 /Frequency

$\Rightarrow$ T = $1/n$

**(5) Relation between velocity, frequency and wavelength :** $v = n\lambda$.

## 10.17 Velocity of Sound (Wave motion)

**(1) Speed of transverse wave motion :**

(i) On a stretched string : $v = \sqrt{\frac{T}{m}}$ , T = Tension in the string;

$m$ = Linear density of string (mass per unit length).

(ii) In a solid body : $v = \sqrt{\frac{\eta}{\rho}}$ ($\eta$ = Modulus of rigidity; $\rho$ = Density of the material.)

**(2) Speed of longitudinal wave motion :**

(i) In a solid long bar $v = \sqrt{\frac{Y}{\rho}}$ (Y = Young's modulus; $\rho$ = Density)

(ii) In a liquid medium $v = \sqrt{\frac{k}{\rho}}$ ($k$ = Bulk modulus)

(iii) In gases $v = \sqrt{\frac{k}{\rho}}$

## 10.18 Velocity of Sound in Elastic Medium

Velocity of sound in any medium is

$v = \sqrt{\frac{E}{\rho}}$ (E = Elasticity of the medium; ρ = Density of the medium)

**(1)** $v_{steel} > v_{water} > v_{air} \Rightarrow 5000$ m/s > 1500 m/s > 330 m/s

**(2) Newton's formula :** He assumed that propagation of sound is isothermal

$v_{air} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{P}{\rho}}$ As $K = E_\theta = P$; $E_\theta$ = Isothermal elasticity; P = Pressure.

By calculation $v_{air}$ = 279 m/sec.

However the experimental value of sound in air is 332 m/sec

**(3) Laplace correction :** He modified that propagation of sound in air is adiabatic process.

$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{E_\phi}{\rho}}$ (As $k = E_\phi = \gamma\rho$ = Adiabatic elasticity)

$v = 331.3$ m/s ($\gamma_{Air} = 1.41$)

**(4) Effect of density :** $v = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow v \alpha \frac{1}{\sqrt{\rho}}$

**(5) Effect of pressure :** Velocity of sound is independent of the pressure (when $T$ = constant)

**(6) Effect of temperature :** $v \alpha \sqrt{T(\text{in K})}$

When the temperature change is small then $v_t = v_0\ (1 + \alpha t)$

Value of $\alpha = 0.608 \frac{m/s}{°C} = 0.61$ (Approx.)

**(7) Effect of humidity :** With rise in humidity velocity of sound increases.

(8) Sound of any frequency or wavelength travels through a given medium ith the same velocity.

(9) Sound of any frequency or wavelength travels through a given medium with the same velocity.

## 10.19 Reflection of Mechanical

| Medium | Longitudinal wave | Transverse wave | Change in direction | Phase change | Time change | Path change |
|---|---|---|---|---|---|---|
| Reflection from rigid end/denser medium | Compression as rarefaction and vice-versa | Crest as crest and Trough as trough | Reversed | $\pi$ | $\frac{T}{2}$ | $\frac{\lambda}{2}$ |
| Reflection from free end/rarer medium | Compression as compression and rarefaction as rarefaction | Crest as trough and trough as crest | No change | Zero | Zero | Zero |

## 10.20 Progressive Wave

(1) These waves propagate in the forward direction of medium with a finite velocity.

(2) Energy and momentum are transmitted in the direction of propagation of waves.

(3) In progressive waves, equal changes in pressure and density occurs at all points of medium.

(4) Various forms of progressive wave function.

(i) $y = A \sin(\omega t - kx)$ Where

$y$ = displacement

$A$ = amplitude

$\omega$ = angular frequency

$n$ = frequency

$k$ = propagation constant

$T$ = time period

$\lambda$ = wave length

$v$ = wave velocity

$t$ = instantaneous time

(ii) $y = A\sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$

(iii) $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

(iv) $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

(v) $y = A\sin\omega\left(t - \frac{x}{v}\right)$

(a) If the sign between $t$ and $x$ terms is negative the wave is propagating along positive X-axis and if the sign is positive then the wave moves in negative X-axis direction.

(b) The Argument of sin or cos function *i.e.* $(\omega t - kx)$ = Phase.

(c) The coefficient of $t$ gives angular frequency

$$\omega = 2\pi n = \frac{2\pi}{T} = vk.$$

(d) The coefficient of $x$ gives propagation constant or wave number

$k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$.

(e) The ratio of coefficient of $t$ to that of $x$ gives wave or phase velocity,

*i.e.* $v = \frac{\omega}{k}$.

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From $v = n\lambda \Rightarrow v \propto \lambda \because n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$.

**(5) Some terms related to progressive waves**

(i) **Wave number** $(\ddot{n})$: The number of waves present in unit length.

$(\ddot{n}) = \frac{1}{\lambda}$.

(ii) Propagation constant $(k)$ : $k = \frac{\phi}{x}$

$$k = \frac{\omega}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \text{ and } k = \frac{2\pi}{\lambda} = 2\pi\bar{\lambda}$$

(iii) Wave velocity ($v$) : $v = \frac{\omega}{k} = n\lambda = \frac{\omega\lambda}{2\pi} = \frac{\lambda}{T}$.

(iv) Phase and phase difference $\phi(x, t) = \frac{2\pi}{\lambda}(vt - x)$.

(v) Phase difference $= \frac{2\pi}{T} \times$ Time difference.

(vi) Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

$\Rightarrow$ Time difference $= \frac{T}{\lambda} \times$ Path difference.

## 10.21 Principle of Superposition

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, .....$ are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement,

$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + .....$

Important applications of superposition principle: (a) Stationary waves, (b) Beats.

## 10.22 Standing Waves or Stationary Waves

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

Characteristics of standing waves :

(1) The disturbance confined to a particular region

(2) There is no forward motion of the disturbance beyond this particular region.

(3) The total energy is twice the energy of each wave.

(4) Points of zero amplitude are known as nodes.

The distance between two consecutive nodes is $\frac{\lambda}{2}$.

(5) Points of maximum amplitude is known as antinodes. The distance between two consecutive antinodes is also $\lambda/2$. The distance between a node and adjoining antinode is $\lambda/4$.

(6) The medium splits up into a number of segments.

(7) All the particles in one segment vibrate in the same phase. Particles in two consecutive segments differ in phase by 180°.

(8) Twice during each vibration, all the particles of the medium pass simultaneously through their mean position.

## 10.23 Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

| S. No. | Parameter | Stretched string | Open organ Pipe | Closed organ Pipe |
|---|---|---|---|---|
| (1) | Fundamental frequency or 1st harmonic (1st mode of vibration) | $n_1 = \frac{v}{2l}$ | $n_1 = \frac{v}{2l}$ | $n_1 = \frac{v}{4l}$ |
| (2) | Frequency of 1st overtone or 2nd harmonic (2nd mode of vibration) | $n_2 = 2n_1$ | $n_2 = 2n_1$ | Missing |
| (3) | Frequency of 2nd overtone or 3rd harmonic (3rd mode of vibration) | $n_3 = 3n_1$ | $n_3 = 3n_1$ | $n_3 = 3n_1$ |
| (4) | Frequency ratio of overtones | 2:3:4: . . . | 2:3:4: . . . | 3:5:7: . . . |
| (5) | Frequency ratio of harmonics | 1:2:3:4: . . . | 1:2:3:4: . . . | 1:3:5:7: . . . |

| | | | |
|---|---|---|---|
| (6) Nature of waves | Transverse stationary | Longitudinal stationary | Longitudinal stationary |
| (7) General formula for wavelength | $\lambda = \frac{2L}{n}, n = 1, 2, 3, ...$ | $\lambda = \frac{2L}{n}, n = 1,$ | $\lambda = \frac{4L}{(2n-1)}$ |
| (8) Position of nodes | $x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} ......$ | $x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} ... \frac{(2n-1)L}{2n}$ | $x = 0, \frac{2L}{(2n-1)}, \frac{4L}{(2n-1)}, \frac{6L}{(2n-1)} .... \frac{2nL}{(2n-1)}$ |
| (9) Position of antinodes | $x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n} .... \frac{(2n-1)L}{2n}$ | $x = \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n} ....$ | $x = \frac{L}{2n-1}, \frac{3L}{2n-1}, \frac{5L}{(2n-1)} ....L$ |

(i) Harmonics are the notes/sounds of frequency equal to or an integral multiple of fundamental frequency ($n$).

(ii) Overtones are the notes/sounds of frequency twice/thrice/ four times the fundamental frequency ($n$).

(iii) In organ pipe an antinode is not formed exactly at the open end rather it is formed a little distance away from the open end outside it. The distance of antinode from the open end of the pipe is = $0.6r$ (where $r$ is radius of organ pipe). This is known as end correction.

## 10.2410 Vibration of a String

General formula of frequency $n_p = \frac{p}{2L}\sqrt{\frac{T}{m}}$

L = Length of string, T = Tension in the string

$m$ = Mass per unit length (linear density), $p$ = mode of vibration

(1) The string will be in resonance with the given body if any of its natural frequencies concides with the body.

(2) If $M$ is the mass of the string of length $L$, $m = \frac{M}{L}$.

So $n = \frac{1}{2Lr}\sqrt{\frac{T}{\pi\rho}}$ ($r$ = Radius, $\rho$ = Density)

## 10.25 Beats

When two sound waves of slightly different frequencies, travelling in a medium along the same direction, superimpose on each other, the intensity of the resultant sound at a particular position rises and falls regularly with time. This phenomenon is called beats.

**(1) Beat period :** The time interval between two successive beats (***i.e.*** two successive maxima of sound) is called beat period.

**(2) Beat frequency :** The number of beats produced per second is called beat frequency.

**(3) Persistence of hearing :** The impression of sound heard by our ears persist in our mind for $1/10^{th}$ of a second.

So for the formation of distinct beats, frequencies of two sources of sound should be nearly equal (difference of frequencies less than 10)

**(4) Equation of beats :** If two waves of equal amplitudes *'a'* and slightly different frequencies $n_1$ and $n_2$ travelling in a medium in the same direction then equation of beats is given by

$y = A \sin \varpi (n_1 - n_2)t$ where $A = 2a \cos \varpi (n_1 - n_2)t$ = Amplitude of resultant wave.

Amplitude of resultant wave.

**(5) Beat frequency :** $n = n_1 \sim n_2$.

**(6) Beat period:** $\dfrac{1}{\text{Beat frequency}} = \dfrac{1}{n_1 \sim n_2}$

## 10.26 Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

Apparent frequency $n' = \dfrac{[(v + v_m) - v_L]n}{[(v + v_m) - v_s]}$

Here $n$ = Actual frequency; $v_L$ = Velocity of listener; $v_s$ = Velocity of source

$v_m$ = Velocity of medium and $v$ = Velocity of sound wave

Sign convention: All velocities along the direction $S$ to $L$ are taken as positive

and all velocities along the direction $L$ to $S$ are taken as negative. If the medium is stationary $v_m = 0$ then $n' = \left(\dfrac{v - v_L}{v - v_s}\right)n.$

(1) No Doppler effect takes place ($n' = n$) when relative motion between source and listener is zero.

(2) Source and listener moves at right angle to the direction of wave propagation. ($n' = n$)

(i) If the velocity of source and listener is equal to or greater than the sound velocity then Doppler effect is not observed.

(ii) Doppler effect does not say about intensity of sound.

(iii) Doppler effect in sound is asymmetric but in light it is symmetric.

# QUESTIONS

## ONE MARK QUESTIONS

**1.** How is the time period effected, if the amplitude of a simple pendulum is increased?

**2.** Define force constant of a spring.

**3.** At what distance from the mean position, is the kinetic energy in simple harmonic oscillator equal to potential energy ?

**4.** How is the frequency of oscillation related with the frequency of change in the K.E. and P.E. of the body in S.H.M.?

**5.** What is the frequency of total energy of a particle in S.H.M. ?

**6.** How is the length of seconds pendulum related with acceleration due gravity of any planet ?

**7.** If the bob of a simple pendulum is made to oscillate in some fluid of density greater than the density of air (density of the bob density of the fluid), then time period of the pendulum increased or decrease.

**8.** How is the time period of the pendulum effected when pendulum is taken to hills or in mines ?

**9.** A transverse wave travels along x-axis. The particles of the medium must move in which direction ?

**10.** Define angular frequency. Give its S.I. unit.

11. Sound waves from a point source are propagating in all directions. What will be the ratio of amplitudes at distances of $x$ meter and $y$ meter from the source ?

12. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position ?

13. What is the time period for the function $f(t) = \sin \omega t + \cos \omega t$ may represent the simple harmonic motion ?

14. When is the swinging of simple pendulum considered approximately SHM ?

15. Can the motion of an artificial satellite around the earth be taken as SHM ?

16. What is the phase relationship between displacement, velocity and acceleration in SHM ?

17. What forces keep the simple pendulum in motion ?

18. How will the time period of a simple pendulum change when its length is doubled ?

19. What is a harmonic wave function ?

20. If the motion of revolving particle is periodic in nature, give the nature of motion or projection of the revolving particle along the diameter.

21. In a forced oscillation of a particle, the amplitude is maximum for a frequency $w_1$ of the force, while the energy is maximum for a frequency $w_2$ of the force. What is the relation between $w_1$ and $w_2$ ?

22. Which property of the medium are responsible for propagation of waves through it ?

23. What is the nature of the thermal change in air, when a sound wave propagates through it ?

24. Why does sound travel faster in iron than in water or air ?

25. When will the motion of a simple pendulum be simple harmonic ?

26. A simple harmonic motion of acceleration '$a$' and displacement '$x$' is represented by $a + 4\pi^2 x = 0$. What is the time period of S.H.M ?

27. What is the main difference between forced oscillations and resonance ?

28. Define amplitude of S.H.M.

**29.** What is the condition to be satisfied by a mathematical relation between time and displacement to describe a periodic motion ?

**30.** Why the pitch of an organ pipe on a hot summer day is higher ?

**31.** Under what conditions does a sudden phase reversal of waves on reflection takes place ?

**32.** The speed of sound does not depend upon its frequency. Give an example in support of this statement.

**33.** If an explosion takes place at the bottom of lake or sea, will the shock waves in water be longitudinal or transverse ?

**34.** Frequency is the most fundamental property of wave, why ?

**35.** How do wave velocity and particle velocity differ from each other ?

**36.** If any liquid of density higher than the density of water is used in a resonance tube, how will the frequency change ?

**37.** Under what condition, the Doppler effect will not be observed, if the source of sound moves towards the listener ?

**38.** What physical change occurs when a source of sound moves and the listener is stationary ?

**39.** What physical change occurs when a source of sound is stationary and the listener moves ?

**40.** If two sound waves of frequencies 480 Hz and 536 Hz superpose, will they produce beats? Would you hear the beats ?

**41.** Define non dissipative medium.

## 2 MARKS QUESTIONS

**42.** Which of the following condition is not sufficient for simple harmonic motion and why ?

(i) acceleration and displacement

(ii) restoring force and displacement

**43.** The formula for time period T for a loaded spring, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

Does the time period depend on length of the spring ?

**44.** Water in a U-tube executes S.H.M. Will the time period for mercury filled up to the same height in the tube be lesser of greater than that in case of water ?

**45.** There are two springs, one delicate and another hard or stout one. For which spring, the frequency of the oscillator will be more ?

**46.** Time period of a particle in S.H.M. depends on the force constant K and mass *m* of the particle $T = 2\pi\sqrt{\frac{m}{k}}$ · A simple pendulum for small angular displacement executes S.H.M. approximately. Why then is the time period of a pendulum independent of the mass of the pendulum ?

**47.** What is the frequency of oscillation of a simple pendulum mounted in a cabin that is falling freely ?

**48.** The maximum acceleration of simple harmonic oscillator is $A_0$. While the maximum velocity is $v_0$, calculate amplitude of motion.

**49.** The velocity of sound in a tube containing air at 27°C and pressure of 76 cm of Hg is 330 $ms^{-1}$. What will be its velocity, when pressure is increased to 152 cm of mercury and temperature is kept constant ?

**50.** Even after the breakup of one prong of tunning fork it produces a round of same frequency, then what is the use of having a tunning fork with two prongs ?

**51.** Why is the sonometer box hollow and provided with holes ?

**52.** The displacement of particle in S.H.M. may be given by $y = a \sin(\omega t + \phi)$ show that if the time $t$ is increased by $2\pi/\omega$, the value of $y$ remains the same.

**53.** The length of simple pendulum executing SHM is increased by 21%. By what % time period of pendulum increase ?

**54.** Define wave number and angular wave number and give their S.l. units.

**55.** Why does the sound travel faster in humid air ?

**56.** Use the formula $v = \sqrt{\frac{\gamma p}{\rho}}$ to explain, why the speed of sound in air

(a) is independent of pressure

(b) increase with temperature

**57.** Differentiate between closed pipe and open pipe at both ends of same length for frequency of fundamental note and harmonics.

**58.** Bats can ascertain distances, directions; nature and size of the obstacle without any eyes, explain how ?

**59.** In a sound wave, a displacement node is a pressure antinode and vice- versa. Explain, why ?

**60.** How does the frequency of a tuning fork change, when the temperature is increased ?

**61.** Explain, why can we not hear an echo in a small room ?

**62.** What do you mean by reverberation? What is reverberation time ?

**3 MARKS QUESTIONS**

**63.** Show that for a particle in linear simple harmonic motion, the acceleration is directly proportional to its displacement of the given instant.

**64.** Show that for a particle in linear simple harmonic motion, the average kinetic energy over a period of oscillation, equals the average potential energy over the same period.

**65.** Deduce an expression for the velocity of a particle executing S.H.M. when is the particle velocity (i) Maximum (ii) minimum?

**66.** Draw (a) displacement time graph of a particle executing SHM with phase angle $\phi$ equal to zero (b) velocity time graph and (c) acceleration time graph of the particle.

**67.** Show that a linear combination of sine and cosine function like $x(t) = a \sin \omega t + b \cos \omega t$ represents a simple harmonic. Also, determine its amplitude and phase constant.

**68.** Show that in a S.H.M. the phase difference between displacement and velocity is $\pi/2$, and between displacement and acceleration is $n$.

**69.** Derive an expression for the time period of the horizontal oscillations of a massless loaded spring.

**70.** Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period.

71. Distinguish with an illustration among free, forced and resonant oscillations.

72. In reference to a wave motion, define the terms

(i) amplitude

(ii) time period

(iii) frequency

(iv) angular frequency

(v) wave length and wave number.

73. What do you understand by phase of a wave? How does the phase change with time and position.

74. At what time from mean position of a body executive S.H.M. kinetic energy and potential energy will be equal?

## LONG ANSWER QUESTIONS

75. Derive expressions for the kinetic and potential energies of a simple harmonic oscillator. Hence show that the total energy is conserved in S.H.M. in which positions of the oscillator, is the energy wholly kinetic or wholly potential ?

76. One end of a U-tube containing mercury is connected to a section pump and the other end is connected to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid in the U-tube executes S.H.M.

77. Discuss the Newton's formula for velocity of sound in air. What correction was applied to it by Laplace and why ?

78. What are standing waves? Desire and expression for the standing waves. Also define the terms node and antinode and obtain their positions.

79. Discuss the formation of harmonics in a stretched string. Show that in case of a stretched string the first four harmonics are in the ratio 1:2:3:4,

80. Give the differences between progressive and stationary waves.

81. If the pitch of the sound of a source appears to drop by 10% to a moving person, then determine the velocity of motion of the person. Velocity of sound = 30 $ms^{-1}$.

82. Give a qualitative discussion of the different modes of vibration of an open organ pipe.

**83.** Describe the various modes of vibrations of a closed organ pipe.

**84.** What are beats? How are they produced? Briefly discuss one application for this phenomenon.

**85.** Show that the speed of sound in air increases by 61 $cms^{-1}$ for every 1°C rise of temperature.

**NUMERICALS**

**86.** The time period of a body executing S.H.M is 1s. After how much time will its displacement be $\frac{1}{\sqrt{2}}$ of its amplitude.

**87.** A particle is moving with SHM in a straight line. When the distance of the particle from the equilibrium position has values $x_1$ and $x_2$, the corresponding value of velocities are $u_1$ and $u_2$. Show that the time period of oscillation is given by

$$t = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

**88.** Find the period of vibrating particle (SHM), which has acceleration of 45 cm $s^{-2}$, when displacement from mean position is 5 cm.

**89.** A 40 gm mass produces on extension of 4 cm in a vertical spring. A mass of 200 gm is suspended at its bottom and left pulling down. Calculate the frequency of its vibration.

**90.** The acceleration due to gravity on the surface of the moon is 1.7 $ms^{-2}$. What is the time period of a simple pendulum on the moon, if its time period on the earth is 3.5 s? [g = 9.8 $ms^{-2}$]

**91.** A particle executes simple harmonic motion of amplitude A.

(i) At what distance from the mean position is its kinetic energy equal to its potential energy?

(ii) At what points is its speed half the maximum speed ?

**92.** A set of 24 tunning forks is arranged so that each gives 4 beats per second with the previous one and the last sounds the octave of first. Find frequency of I$^{st}$ and last tunning forks.

**93.** The vertical motion of a huge piston in a machine is approximately S.H.M.

with a frequency of 0.5 $s^{-1}$. A block of 10kg is placed on the piston. What is the maximum amplitude of the piston's S.H.M. for the block and piston to remain together ?

**94.** At what temperature will the speed of sound be double its value at 273°K?

**95.** A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60 s. What is the weight of the body ?

**96.** If the pitch of the sound of a source appears to drop by 10% to a moving person, then determine the velocity of motion of the person. Velocity of sound = 330 $ms^{-1}$.

**97.** A body of mass *m* suspended from a spring executes SHM. Calculate ratio of K.E. and P.E. of body when it is at a displacement half of its amplitude from mean position.

**98.** A string of mass 2.5 kg is under a tension of 200N. The length of the stretched string is 20m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end ?

**99.** Which of the following function of time represent (a) periodic and (*n*) non-periodic motion? Give the period for each case of periodic motion. [*w* is any positive constant].

(i) $\sin \omega t + \cos \omega t$

(ii) $\sin \omega t + \sin 2\omega t + \sin 4 \omega t$

(iii) $e^{-\omega t}$

(iv) $\log (\omega t)$

**100.** The equation of a plane progressive wave is given by the equation $y = 10 \sin 2\pi (t - 0.005x)$ where $y$ and $x$ are in cm and $t$ in seconds. Calculate the amplitude, frequency, wave length and velocity of the wave.

**101.** A tuning fork arrangement (pair) produces 4 beats $s^{-1}$ with one fork of frequency 288 cps. A little was ix placed on the unknown fork and it then produces 2 beats $s^{-1}$. What is the frequency of the unknown fork ?

**102.** A pipe 20 cm long is closed at one end, which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will this same source can be in resonance with the pipe, if both ends are open? Speed of sound = 340 $ms^{-1}$.

**103.** The length of a wire between the two ends of a sonometer is 105 cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1 : 3 : 15 ?

**104.** The transverse displacement of a string (clamped at its two ends) is given by

$$y(x,t) = 0.06 \sin \frac{2\pi}{3} \cos 120\,\omega t.$$

where $x$, $y$ are in m and $t$ is in s. The length of the string is 1.5 m and its mass is $3.0 \times 10^{-2}$ kg. Answer the following.

(a) Does the function represent a travelling or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength frequency and speed of propagation of each wave ?

(c) Determine the tension in the string.

**105.** A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency 45 Hz. The mass of the wire is $3.5 \times 10^{-2}$ kg and its linear density is $4.0 \times 10^{-2}$ kg m$^{-1}$. What is (a) the speed of transverse wave on the string and (b) the tension in the string ?

**106.** A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod as given to be 2.53 kHz. What is the speed of sound in steel ?

**107.** A progressive wave of frequency 500 Hz is travelling with velocity 360 m/s. How far apart are two points 60° out of phase ?

**108.** An observer moves towards a stationary source of sound with a velocity one fifth of velocity of sound. What is the % increase in apparent frequency ?

## SOLUTIONS

### ANSWERS OF ONE MARK QUESTIONS

**1.** No effect on time period when amplitude of pendulum is increased or decreased.

**2.** The spring constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring. ($K = f/x$)

**3.** At $x = a/\sqrt{2}$ ., KE = PE = $\frac{1}{4} m\omega^2 a^2$

4. P.E. or K.E. completes two vibrations in a time during which S.H.M. completes one vibration or the frequency of R.E. or K.E is double than that of S.H.M.

5. The frequency of total energy of particle is S.H.M. is zero because it remains constant.

6. Length of the seconds pendulum proportional to (acceleration due to gravity)

7. Increased

8. As $T \mu \frac{1}{\sqrt{g}}, T$ will increase.

9. In the *y-z* plane or in plane perpendicular to *x*-axis.

10. It is the angle covered per unit time or it is the quantity obtained by multiplying frequency by a factor of $2\pi$.

    $\omega = 2\pi n$, S.I. unit is rad $s^{-1}$.

11. Intensity = amplitude$^2 \propto \frac{1}{(\text{distance})^2}$

    ∴ Required ratio = $y/x$

12. No, the resultant of Tension in the string and weight of bob is not always towards the mean position.

13. $T = 2\pi/\omega$

14. Swinging through small angles.

15. No, it is a circular and periodic motion but not SHM.

16. In SHM, The velocity leads the displacement by a phase $\pi/2$ radians and acceleration leads the velocity by a phase $\pi/2$ radians.

17. The component of weight ($mg \sin \theta$).

18. $\sqrt{2}$ times, as $T \alpha \sqrt{l}$

19. A harmonic wave function is a periodic function whose functional form is sine or cosine.

20. S.H.M.

21. Both amplitude and energy of the particle can be maximum only in the case of resonance, for resonance to occur $\omega_1 = \omega_2$.

22. Properties of elasticity and inertia.

23. When the sound wave travel through air adiabatic changes take place in the medium.

24. Sound travel faster in iron or solids because iron or solid is highly elastic as compared to water (liquids) or air (gases).

25. When the displacement of bob from the mean position is so small that $\sin\theta \approx \theta$.

26. $a = -4\pi^2 x = -\omega^2 x \Rightarrow \omega = 2\pi$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1s$$

27. The frequency of external periodic force is different from the natural frequency of the oscillator in case of forced oscillation but in resonance two frequencies are equal.

28. The maximum displacement of oscillating particle on either side of its mean position is called its amplitude.

29. A periodic motion repeats after a definite time interval $T$.

So, $y(t) = y(t + T) = y(t + 2T)$ etc.

30. On a hot day, the velocity of sound will be more since (frequency proportional to velocity) the frequency of sound increases and hence its pitch increases.

31. On reflection from a denser medium, a wave suffers a sudden phase reversal.

32. If sounds are produced by different musical instruments simultaneously, then all these sounds are heard at the same time.

33. Explosion at the bottom of lake or sea create enormous increase in pressure of medium (water). A shock wave is thus a longitudinal wave travelling at a speed which is greater than that of ordinary wave.

34. When a wave passes through different media, velocity and wavelength change but frequency does not change.

35. Wave velocity is constant for a given medium and is given by $V = n\lambda$. But particle velocity changes harmonically with time and it is maximum at mean position and zero at extreme position.

36. The frequency of vibration depends on the length of the air column and not on reflecting media, hence frequency does not change.

37. Doppler effect will not be observed, if the source of sound moves towards the listener with a velocity greater than the velocity of sound. Same is also true if listener moves with velocity greater than the velocity of sound towards the source of sound.

38. Wave length of sound changes.

39. The number of sound waves received by the listener changes.

40. Yes, the sound waves will produce 56 beats every second. But due to persistence of hearing, we would not be able to hear these beats.

41. A medium in which speed of wave motion is independent of frequency of wave is called non-dispersive medium. For sound, air is non dispersive medium.

## ANSWERS OF TWO MARKS QUESTIONS

42. Condition (i) is not sufficient, because direction of acceleration is not mentioned. In SHM, the acceleration is always in a direction opposite to that of the displacement.

43. Although length of the spring does not appear in the expression for the time period, yet the time period depends on the length of the spring. It is because, force constant of the spring depends on the length of the spring.

44. The time period of the liquid in a U-tube executing S.H.M. does not depend upon density of the liquid, therefore time period will be same, when the mercury is filled up to the same height in place of water in the U-tube.

45. We have, $v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$

So, when a hard spring is loaded with a mass $m$. The extension I will be lesser w.r.t. delicate one. So frequency of the oscillation of the hard spring will be more and if time period is asked it will be lesser.

46. Restoring force in case of simple pendulum is given by

$$F = \frac{mg}{l}y \Rightarrow K = mg/l$$

So force constant itself proportional to $m$ as the value of $k$ is substituted in the formula, $m$ is cancelled out.

**47.** The pendulum is in a state of weightlessness *i.e.* $g = 0$. The frequency of pendulum

$$v = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = 0$$

**48.** $A_{max} = \omega^2 a = A_0$, $U_{max} = \omega a = v_0$

$$\Rightarrow \omega = \frac{v_0}{a}$$

$$\therefore \qquad a = \frac{A_0}{\omega^2} \qquad \Rightarrow a = \frac{A_0}{v_0^2}a^2$$

$$\Rightarrow a = \frac{v_0^2}{A_0}.$$

**49.** At a given temperature, the velocity of sound is independent of pressure, so velocity of sound in tube will remain 330 $ms^{-1}$.

**50.** Two prongs of a tunning fork set each other in resonant vitorations and help to maintain the vibrations for a longer time.

**51.** When the stem of the a tunning fork gently pressed against the top of sonometer box, the air enclosed in box also vibrates and increases the intensity of sound. The holes bring the inside air incontact with the outside air and check the effect of elastic fatigue.

**52.** The displacement at any time $t$ is

$$y = a \sin(\omega t + \phi)$$

$\therefore$ displacement at any time $(t + 2\pi/\omega)$ will be

$$y = a \sin[\omega(t + 2\pi/\omega) + \phi] = [\sin\{\omega t + \phi) + 2\pi\}]$$

$$y = a \sin(\omega t + \phi) \qquad [\because \sin(2\pi + \phi) = \sin\phi]$$

Hence, the displacement at time $t$ and $(t + 2\pi/\omega)$ are same.

**53.** When a number of waves travel through the same region at the same time, each wave travels independently as if all other waves were absent.

This characteristic of wave is known as independent behaviour of waves. For example we can distinguish different sounds in a full orchestra.

**54.** Wave number is the number of waves present in a unit distance of medium. $(\bar{v} = 1/\lambda)$ S.I. unit of $k$ is rad m$^{-1}$.

Angular wave number or propagation constant is $2\pi/\lambda$. It represents phase change per unit path difference and denoted by $k = 2\pi/\lambda$. S.I. unit of $k$ is rad m$^{-1}$.

**55.** Because the density of water vapour is less than that of the dry air hence density of air decreases with the increase of water vapours or humidity and velocity of sound inversely proportional to square root of density.

**56.** Given, $v = \sqrt{\dfrac{\gamma P}{\rho}}$

(a) Let V be the volume of 1 mole of air, then

$$\rho = \frac{M}{V} \quad \text{or} \quad V = \frac{M}{\rho}$$

for 1 mole of air PV = RT

$$\therefore \quad \frac{PM}{\rho} = RT \quad \text{or} \quad \frac{P}{\rho} = \frac{RT}{M}$$

$$\Rightarrow \quad v = \sqrt{\frac{\gamma RT}{M}} \qquad \text{....(i)}$$

So at constant temperature $v$ is constant as $\gamma$, R and M are constant.

(b) From equation (i) we know that $V \alpha \sqrt{T}$, so with the increase in temperature velocity of sound increases.

**57.** (i) In a pipe open at both ends, the frequency of fundamental note produced is twice as that produced by a closed pipe of same length.

(ii) An open pipe produces all the harmonics, while in a closed pipe, the even harmonics are absent,

**58.** Bats emit ultrasonic waves of very small wavelength (high frequencies) and so high speed. The reflected waves from an obstacle in their path give them idea about the distance, direction, nature and size of the obstacle.

**59.** At the point, where a compression and a rarefaction meet, the displacement is minimum and it is called displacement node. At this point, pressure difference is maximum *i.e.* at the same point it is a pressure antinode. On the other hand, at the mid point of compression or a rarefaction, the displacement variation is maximum i.e. such a point is pressure node, as pressure variation is minimum at such point.

**60.** As the temperature increases, the length of the prong of the tunning fork increases. This increases the wavelength of the stationary waves set up in the tunning fork. As frequency, $v \propto \sqrt{\frac{1}{\lambda}},$ so frequency of the tunning fork decreases.

**61.** For an echo of a simple sound to be heard, the minimum distance between the speaker and the walls should be 17 m, so in any room having length less than 17 m, our ears can not distinguish between sound received directly and sound received after reflection.

**62.** The phenomenon of persistence or prolongation of sound after the source has stopped emitting sound is called reverberation. The time for which the sound persists until it becomes inaudible is called the reverberation time.

**SOLUTION / HINTS OF NUMERICALS**

**86.** $$y = r \sin \omega t = r \sin \frac{2\pi}{T} t$$

Here $y = \frac{1}{3} r$ and $T = 1s$

$$\therefore \quad \frac{1}{\sqrt{2}} r = r \sin \frac{2\pi}{T} t \Rightarrow 2\pi t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{1}{8} s.$$

**87.** When $x = x_1, v = u_1$

When $x = x_2, v = u_2$

As $v = \omega\sqrt{A^2 - x^2}$

$$\therefore \quad u_1 = \omega\sqrt{A^2 - x_1^2} \quad \text{or} \quad u_1^2 = \omega^2(A^2 - x_1^2) \qquad ....(i)$$

and $u_2 = \omega\sqrt{A^2 - x_2^2}$ or $u_2^2 = \omega^2(A^2 - x_2^2)$ ....(ii)

Subtracting (ii) from (i), we get

$$u_1^2 - u_2^2 = (A^2 - x_1^2) - \omega^2(A^2 - x_2^2) = \omega^2(x_2^2 - x_1^2)$$

or $$\omega = \left[\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}\right]^{1/2}$$

$$T = \frac{2\pi}{\omega} = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

**88.** Here $y = 5$ cm and acceleration $a = 45$ cm $s^{-2}$

We know $a = \omega^2 y$

$\therefore\ 45 = \omega^2 \times 5$ or $\omega = 3$ rad $s^{-1}$

and $T = \frac{2\pi}{\omega} = \frac{2\pi}{3} = 2.095s.$

**89.** Here $mg = 40\ g = 40 \times 980$ dyne ; $l = 4$ cm.

say $k$ is the force constant of spring, then

$mg = kl$ or $k = mg/l$

$$k = \frac{40 \times 980}{4} = 9800 \text{ dyne cm}^{-1}$$

when the spring is loaded with mass $m = 200$ g

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9800}{200}}$$

$= 1.113\ s^{-1}$.

**90.** Here on earth, $T = 3.5\ s;\ g = 9.8\ ms^{-2}$

For simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

$$3.5 = 2\pi\sqrt{\frac{l}{9.8}} \quad \text{....(i)}$$

on moon, $g' = 1.7$ ms$^{-2}$ and if $T'$ is time period

then $T' = 2\pi\sqrt{\frac{l}{1.7}}$ ....(ii)

Dividing eqn. (ii) by eqn. (i), we get

$$\frac{T'}{3.5} = \sqrt{\frac{9.8}{1.7}} \quad \text{or } T' = \sqrt{\frac{9.8}{1.7}} \times 3.5 = 8.4s$$

**91.** (i) $\frac{1}{2}m\omega^2(a^2 - y^2) = \frac{1}{2}m\omega^2 y^2 \Rightarrow y = \frac{a}{\sqrt{2}}$

(ii) $v = \omega\sqrt{a^2 - y^2} \Rightarrow a\omega = \omega\sqrt{a^2 - y^2} \Rightarrow y = \pm\frac{a\sqrt{3}}{2}$.

**92.** Let frequency of $I^{st}$ tunning fork = $x$

frequency of $II^{nd}$ tunning fork = $x + 4$

frequency of $III^{rd}$ tunning fork = $x + 2$ (4)

frequency of $IV^{th}$ tunning fork = $x + 3$ (4)

∴ Let frequency of $24^{th}$ tunning fork = $x + 23$ (4)

octave means, (twice in freq.)

∴ freq. of 24th = 2 × freq. of $I^{st}$ = $2x$

∴ $2x = x + 23\ (4) \Rightarrow x = 92$

freq. of $24^{th}$ = 2 × 92 = 184 $H_3$.

**93.** Given, $v = 0.5\ s^{-1}$, $g = 9.8\ ms^{-1}$

$$a = \omega^2 y = (2\pi v)^2 y = 4\pi^2 v^2 y$$

$a_{max}$ at the extreme position *i.e.*, $r = y$

$a_{max} = 4\pi^2 v^2 r$ and $a_{max} = g$ to remain in contact.

or $$r = \frac{g}{4\pi^2 v^2} = \frac{9.8}{4\pi^2 \times (0.5)^2} = 0.993\text{m}.$$

**94.** Say $v_1$ in the velocity of sound at $T_1 = 273°K$ and $v_2 = 2v_1$ at temperature $T_2$

Now $\frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$, $\therefore \frac{2v_1}{v_1} = \sqrt{\frac{T_2}{273}}$

or $T_2 = 4 \times 273 = 1092°K$.

**95.** Here $m = 50$ kg, $l = 0.2\ m$

we know $mg = kl$ or $k = \frac{mg}{l} = \frac{50 \times 9.8}{0.2} = 2450 \text{ Nm}^{-1}$

$T = 0.60\ s$ and $M$ is the mass of the body, then using

$$T = 2\pi\sqrt{\frac{M}{k}} \Rightarrow M = \frac{2450 \times (0.60)^2}{4\pi^2} = 22.34 \text{ kg}$$

Weight of body Mg = 22.34 × 9.8 = 218.93 *N*.

**96.** Apparent freq.

$$v' = \left(\frac{v - v_0}{v}\right)v \quad \text{or} \quad \frac{v'}{v} = \frac{v - v_0}{v}$$

$$\frac{v'}{v} = \frac{900}{1000} = \frac{9}{10}, \quad v = 330 \text{ ms}^{-1}$$

$$\therefore \quad \frac{9}{10} = \frac{330 - v_0}{330}$$

$$330 - v_0 = \frac{9}{10} \times 330 = 297$$

$$v_0 = 330 - 297 = 33 \text{ m/s}.$$

**97.**

$$\text{KE} = \frac{1}{2}m\omega^2(a^2 - y^2)$$

at $y = \frac{a}{2}$

$$\text{KE} = \frac{1}{2}m\omega^2\left[a^2 - \left(\frac{a}{2}\right)^2\right] = \frac{1}{2}m\omega^2 . \frac{3a^2}{4}$$

$$\text{PE} = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2\frac{a^2}{4}$$

$$\frac{\text{KE}}{\text{PE}} = \frac{3}{1}.$$

**98.** Given $T = 200$ N, length of string $l = 20$ m

total mass of the string = 2.5 kg

∴ mass per unit length of the string

$$m = \frac{2.5}{20} = 0.125 \text{ kg } m^{-1}$$

Now $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{200}{0.125}} = 40 \text{ ms}^{-1}$

Hence time taken by the transverse wave to reach other end

$$t = \frac{l}{v} = \frac{20}{40} = 0.5s.$$

**99.** (i) $\sin \omega t + \cos \omega t = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin \omega t + \frac{1}{\sqrt{2}}\cos \omega t\right]$

$$= \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$$

It is simple harmonic function with period $= \frac{2\pi}{\omega}$

(ii) $\sin \omega t + \sin 2\omega t + \sin 4\omega t$ is a periodic but not simple harmonic function. Its time period is $\frac{2\pi}{\omega}$.

(iii) $e^{-\omega t}$ is exponential function, which never repeat itself. Hence it is non-periodic function.

(iv) $\log \omega t$ is also non-periodic function.

**100.** Here $y = 10\sin 2\varpi(t - 0.005x)$

$$y = 10\sin\frac{2\pi}{200}(200t - x) \quad \text{....(i)}$$

The equation of a travelling wave is given by

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \quad \text{....(ii)}$$

Comparing the equation (i) and (ii), we have

$\alpha = 10$ cm, $\lambda = 200$ cm and $v = 200 \text{ ms}^{-1}$

Now $v = \frac{v}{\lambda} = \frac{200}{200} = 1 \text{ Hz}$

**101.** Unknown freq. = Known freq. I Beat freq.

= 288 ± 4 = 292 or 284 Hz

On putting wax, freq. decreases, beat freq. is also descrease to 2

$\therefore$ unknown freq. = 292 Hz (higher one)

**102.** The frequency of $n^{th}$ mode of vibration of a pipe closed at one end is given by

$$v_n = \frac{(2n-1)v}{4L}$$

river $v = 340$ ms$^{-1}$, $L = 20$ cm = 0.2 m; $v_n = 430$ Hz

$$\therefore \qquad 430 = \frac{(2n-1)\times 340}{4\times 0.2} \Rightarrow n = 1$$

Therefore, first mode of vibration of the pipe is excited, for open pipe since $n$ must be an integer, the same source can not be in resonance with the pipe with both ends open.

**103.** Total length of the wire, $L = 105$ cm

$$v_1 : v_2 : v_3 = 1 : 3 : 15$$

Let $L_1$, $L_2$ and $L_3$ be the length of the three parts. As $v \propto \frac{1}{L}$

$$\therefore \quad L_1 : L_2 : L_3 = 1 : \frac{1}{3} : \frac{1}{15} = 15 : 5 : 1$$

Sum of the ratios = 15 + 5 + 1 = 21

$$\therefore \qquad L_1 = \frac{15}{21}\times 105 = 75 \text{ cm}; \; L_2 = \frac{5}{21}\times 105 = 25 \text{ cm};$$

$$L_3 = \frac{1}{21}\times 105 = 5 \text{ cm}$$

Hence the bridges should be placed at 75 cm and (75 + 25) = 100 cm from one end.

**104.** $y(x,t) = 0.06 \sin \frac{2\pi}{3} \times \cos 120\,\pi t$ ....(i)

(a) The displacement which involves harmonic functions of $x$ and $t$ separately represents a stationary wave and the displacement, which is harmonic function of the form $(vt \pm x)$, represents a travelling wave. Hence, the equation given above represents a stationary wave.

(b) When a wave pulse $y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$ travelling along $x$-axis is superimposed by the reflected pulse.

$y_2 = -a\sin\frac{2\pi}{\lambda}(vt + x)$ from the other end, a stationary wave is formed and is given by

$$y = y_1 + y_2 = -2a\sin\frac{2\pi}{\lambda} \times \cos\frac{2\pi}{\lambda}vt \qquad \text{....(ii)}$$

Comparing the eqs. (i) and (ii), we have

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \quad \text{or} \quad \lambda = 3\text{m}$$

$$\frac{2\pi}{\lambda}v = 120\pi \quad \text{or} \quad v = 60\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$$

Now frequency $\gamma = \frac{v}{\lambda} = \frac{180}{3} = 60$ Hz

(c) Velocity of transverse wave in a string is given by

$$v = \sqrt{\frac{T}{m}}$$

Here $\quad m = \frac{3\times10^{-2}}{1.5} = 2\times10^{-2}$ kgm$^{-1}$

Also $\quad v = 180$ ms$^{-1}$

$\therefore \quad T = v^2 m = (180)^2 \times 2 \times 10^{-2} = 648\text{N}.$

**105.** Frequency of fundamental mode, $v = 45$Hz

Mass of wire $M = 3.5 \times 10^{-2}$ kg; mass per unit length, $m = 4.0 \times 10^{-2}$ kgm$^{1}$

$\therefore \quad$ Length of wire $L = \frac{M}{m} = \frac{3.5\times10^{-2}}{4.0\times10^{-2}} = 0.875\,m$

(a) For fundamental mode $L = \frac{\lambda}{2}$ or $\lambda = 2L = 0.875 \times 2 = 1.75$ m

$\therefore$ velocity $v = v\lambda = 45 \times 1.75 = 78.75$ ms$^{-1}$

(b) The velocity of transverse wave

$$v = \sqrt{\frac{T}{m}} \Rightarrow T = v^2 m = (78.75)^2 \times 4.0 \times 10^2 = 248.6\,N$$

**106.** Given : $u = 2.53$ kHz $= 2.53 \times 10^3$ Hz

(L) Length of steel rod = 100 cm = 1 m.

when the steel rod clamped at its middle executes longitudinal vibrations of its fundamental frequency, then

$$L = \frac{\lambda}{2} \text{ or } \lambda = 2L = 2 \times 1 = 2 \text{ m}$$

The speed of sound in steel

$$v = n\lambda = 2.53 \times 10^3 \times 2 = 5.06 \times 10^3 \text{ ms}^{-1}.$$

**107.** $\Delta\phi = 60° = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ rad.

$$v = v\lambda \Rightarrow \lambda = \frac{v}{v} = \frac{360}{500} = 0.72 \text{ m}$$

As $\Delta\phi = \frac{2\pi}{\lambda}.\Delta x$

$$\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi = \frac{\pi}{3} \times \frac{0.72}{2\pi}$$

$= 0.12$ m.

**108.** $v_0 = -\frac{v}{5}, v_s = 0$

Apparent freq. $v' = \left(\frac{v - v_0}{v - v_s}\right) v$

$$= \left(\frac{v + \frac{v}{5}}{v - 0}\right) v = \frac{6}{5} v = 1.2v$$

$$\% \text{ change} = \frac{\Delta v}{v} \times 100 = \frac{1.2v - v}{v} \times 100 = 20\%$$

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