Unit - 1

Dimensions And Measurement

1.1 Physical Quantity

A quantity which can be measured and expressed in form of laws is called a physical quantity. Physical quantity (Q) = Magnitude × Unit = $n \times u$

Where, n represents the numerical value and u represents the unit. as the unit(u) changes, the magnitude (n) will also change but product 'nu' will remain same.

i.e. $n\,u$ = constant, or $n_1u_1 = n_2u_2$ = constant;

1.2 Fundamental and Derived Units

Any unit of mass, length and time in mechanics is called a *fundamental, absolute or base unit.* Other units which can be expressed in terms of fundamental units, are called derived units

System of units : A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units.

(1) CGS system, (2) MKS system, (3) FPS system.

(4) **S.I. system :** It is known as International system of units. There are seven fundamental quantities in this system. These quantities and their units are given in the following table.

Quantity	Name of Units	Symbol
Length	Metre	m
Mass	Kilogram	kg
Time	Second	*s*
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	Mol
Luminous Intensity	Candela	Cd

Besides the above seven fundamental units two supplementary units are also defined - Radian (*rad*) for plane angle and Steradian (*sr*) for solid angle.

1.3 Dimensions of a Physical Quantity

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

1.4 Important Dimensions of Complete Physics

Mechanics

S.N.	Quantity	Unit	Dimension
(1)	Velocity or speed (v)	m/s	$[M^0L^1T^{-1}]$
(2)	Acceleration (a)	m/s^2	$[M^0LT^{-2}]$
(3)	Momentum (P)	kg.m/s	$[M^1L^1T^{-1}]$
(4)	Impulse (I) kg. m/s	Newton sec or	$[M^1L^1T^{-1}]$
(5)	Force (F)	Newton	$[M^1L^1T^{-2}]$
(6)	Pressure (P)	Pascal	$[M^1L^{-1}T^{-2}]$
(7)	Kinetic energy (E_k)	Joule	$[M^1L^2T^{-2}]$
(8)	Power (P)	Watt or Joule/s	$[M^1L^2T^{-3}]$
(9)	Density (d)	kg/m^3	$[M^1L^{-3}T^0]$
(10)	Angular displacement (θ)	Radian (rad.)	$[M^0L^0T^0]$
(11)	Angular velocity (ω)	Radian/sec	$[M^0L^0T^{-1}]$
(12)	Angular Acceleration (α)	$Radian/sec^2$	$[M^0L^0T^{-2}]$
(13)	Moment of inertia (I)	$kg.m^2$	$[ML^2T^0]$
(14)	Torque (τ)	Newton-meter	$[ML^2T^{-2}]$
(15)	Angular momentum (L)	Joule sec	$[ML^2T^{-1}]$
(16)	Force constant or spring constant (k)	Newton/m	$[M^1L^0T^{-2}]$
(17)	Gravitational constant (G)	$N–m^2/kg^2$	$[M^{-1}L^3T^{-2}]$

(18)	Intensity of gravitational field (E_g)	N/kg	$[M^0L^1T^{-2}]$
(19)	Gravitational potential (V_g)	Joule/kg	$[M^0L^2T^{-2}]$
(20)	Surface tension (T)	N/m or Joule/m^2	$[M^1L^0T^{-2}]$
(21)	Velocity gradient (V_g)	$Second^{-1}$	$[M^0L^0T^{-1}]$
(22)	Coefficient of viscosity (η)	kg/m s	$[M^1L^{-1}T^{-1}]$
(23)	Stress	N/m^2	$[M^1L^{-1}T^{-2}]$
(24)	Strain	No unit	$[M^0L^0T^0]$
(25)	Modulus of elasticity (E)	N/m^2	$[M^0L^{-1}T^{-2}]$
(26)	Poisson Ratio (σ)	No unit	$[M^0L^0T^0]$
(27)	Time period (T)	Second	$[M^0L^0T^1]$
(28)	Frequency (n)	Hz	$[M^0L^0T^{-1}]$

Heat

S.N.	Quantity	Unit	Dimension
(1)	Temperature (T)	Kelvin	$M^0L^0T^0K^1]$
(2)	Heat (Q)	Joule	$[ML^2T^{-2}]$
(3)	Specific Heat (c)	Joule/Kg–K	$[M^0L^2T^{-2}K^{-1}]$
(4)	Thermal capacity	Joule/K	$[M^1L^2T^{-2}K^{-1}]$
(5)	Latent heat (L)	Joule/kg	$[M^0L^2T^{-2}]$
(6)	Gas constant (R)	Joule/mol-K	$[M^1L^2T^{-2}mol^{-1}K^{-1}]$
(7)	Boltzmann constant (k)	Joule/K	$[M^1L^2T^{-2}K^{-1}]$
(8)	Coefficient of thermal conductivity (K)	Joule/M-s-K	$[M^1L^1T^{-3}K^{-1}]$
(9)	Stefan's constant (σ)	Watt/m^2–K^4	$[M^1L^0T^{-3}K^{-4}]$
(10)	Wien's constant (b)	Meter K	$[M^0L^1T^0K^1]$
(11)	Planck's constant (h)	Joule s	$[M^1L^2T^{-1}]$
(12)	Coefficient of Linear Expansion	$Kelvin^{-1}$	$[M^0L^0T^0K^{-1}]$

(13)	Mechanical eq. of Heat (J)	Joule/Calorie	$[M^0L^0T^0]$
(14)	Vander wall's constant (*a*)	Newton m^4	$[M^1L^5T^{-2}]$
(15)	Vander wall's consatnt (*b*)	m^3	$[M^0L^3T^0]$

1.5 Quantities Having Same Dimensions

S.N.	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^0T^{-2}K^{-1}]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g},$ *g* = acceleration due to gravity, *m* = mass, *k* = spring constant
(13)	$[M^0L^0T^1]$	L/R $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance
(14)	$[ML^2T^{-2}]$	I^2Rt, $\frac{V^2}{R}t$, VIt, qV, LI^2, $\frac{q^2}{C}$, CV^2 where I = current, *t* = time *q* = charge, L = inductance, C = capacitance, R = resistance

1.6 Application of Dimensional Analysis.

(1) To find the unit of a physical quantity in a given system of units.

(2) To find dimensions of physical constant or coefficients.

(3) To convert a physical quantity from one system to the other.

(4) To check the dimensional correctness of a given physical relation: This is based on the *'principle of homogeneity'*. According to this principle the dimensions of each term on both sides of an equation must be the same.

(5) To derive new relations.

1.7 Limitations of Dimensional Analysis.

(1) If dimensions are given, physical quantity may not be unique.

(2) Numerical constant having no dimensions cannot be deduced by the methods of dimensions.

(3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$s = u\,t + (1/2)\,at^2$ or $y = a \sin \omega\, t$

(4) The method of dimensions cannot be applied to derive formula consist of more than 3 physical quantities.

1.8 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

(1) All non-zero digits are significant.

(2) A zero becomes significant figure if it appears between two non-zero digits.

(3) Leading zeros or the zeros placed to the left of the number are never significant.

Example : 0.543 has three significant figures.
0.006 has one significant figures.

(4) Trailing zeros or the zeros placed to the right of the number are significant.

Example : 4.330 has four significant figures.
343.000 has six significant figures.

(5) In exponential notation, the numerical portion gives the number of significant figures.

Example : 1.32×10^{-2} has three significant figures.

1.9 Rounding Off

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example : $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ rounded off to 3.9.

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example : $x = 6.87$ is rounded off to 6.9, again $x = 12.78$ is rounded off to 12.8.

(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.

Example : $x = 16.351$ is rounded off to 16.4, again $x = 6.758$ is rounded off to 6.8.

(4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.

Example : $x = 3.250$ becomes 3.2 on rounding off, again $x = 12.650$ becomes 12.6 on rounding off.

(5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

Example : $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

1.10 Significant Figures in Calculation

The following two rules should be followed to obtain the proper number of significant figures in any calculation.

(1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places.

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation.

1.11 Order of Magnitude

Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one.

For example,

(1) Speed of light in vacuum = 3×10^8 ms^{-1} $\approx 10^8$ m/s (ignoring $3 < 5$)

(2) Mass of electron = 9.1×10^{-31} kg 10^{-30} kg (as $9.1 > 5$).

1.12 Errors of Measurement.

The measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(1) Absolute error—Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be $a_1, a_2, a_3, \ldots a_n$. The arithmetic mean of these value is $a_m = \frac{a_1 + a_2 + \ldots a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_1$$
$$\ldots\ldots\ldots\ldots$$
$$\Delta a_n = a_m - a_2$$

The absolute errors may be positive in certain cases and negative in certain other cases.

(2) Mean absolute error—It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by Δa. Thus

$$\Delta a = \frac{|\Delta a_1| + |\Delta a_2| + \ldots |\Delta a_n|)}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between

$(a_m - \overline{\Delta a})$ and $(a_m + \overline{\Delta a})$.

(3) Relative error or Fractional error—Relative error or Fractional error

$$= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}.$$

(4) Percentage error : Percentage error $= \frac{\overline{\Delta a}}{a_m} \times 100\%$.

1.13 Propagation of Errors

(1) Error in sum of tie quantities : Suppose $x = a + b$

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x *i.e.* sum of a and b.

The maximum absolute error in x is $\Delta x = \pm (\Delta a + \Delta b)$

(2) Error in difference of the quantities—Suppose $x = a - b$

The maximum absolute error in x is $\Delta x = \pm (\Delta a + \Delta b)$

(3) Error in product of quantities—Suppose $x = a \times b$

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(4) Error in division of quantities—Suppose $x = \frac{a}{b}$

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

(5) Error in quantity raised to some power—Suppose $x = \frac{a^n}{b^m}$

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

- The quantity which have maximum power must be measured carefully because it's contribution to error is maximum.

UNIT I – UNITS & MEASUREMENT

1. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit if light takes 8 min and 20 s to cover this distance.

2. If $x = a + bt + ct^2$, where x is in metre and t in seconds, what is the unit of c ?

3. What is the difference between mN, Nm and nm ?

4. The radius of atom is of the order of 1Aº & radius of Nucleus is of the order of fermi. How many magnitudes higher is the volume of the atom as compared to the volume of nucleus ?

5. How many kg make 1 unified atomic mass unit ?

6. Name same physical quantities that have same dimension.

7. Name the physical quantities that have dimensional formula $[ML^{-1}T^{-2}]$.

8. Give two examples of dimension less variables.

9. State the number of significant figures in

(i) 0.007 m^2	(ii) 2.64×10^{24} kg
(iii) 0.2370 g cm^{-3}	(iv) 0.2300m
(v) 86400	(vi) 86400 m

10. Given relative error in the measurement of length is .02, what is the percentage error ?

11. A physical quantity P is related to four observables a, b, c and d as follows :

$$P = a^3b^2/(\sqrt{cd})$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P? What is the value of P calculated using the above relation turns out to be 3.763, to value should you round off the result ?

12. A boy recalls the relation for relativistic mass (m) in terms of rest mass (m_0) velocity of particle V, but forgets to put the constant c (velocity of light). He writes $m = \dfrac{m_0}{(1-v^2)^{1/2}}$ correct the equation by putting the missing 'c'.

13. Name the technique used in locating.

(a) an under water obstacle

(b) position of an aeroplane in space.

14. Deduce dimensional formulae of—

(i) Boltzmann's constant

(ii) mechanical equivalent of heat.

15. Give examples of dimensional constants and dimensionless constants.

SHORT ANSWER QUESTIONS (2 MARKS)

16. The vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm. Calculate the minimum inaccuracy in the measurement of distance.

17. If the unit of force is 100N, unit of length is 10m and unit of time is 100s. What is the unit of Mass in this system of units ?

18. Describe the principle and use of SONAR and RADAR.

19. State the principle of homogeneity. Test the dimensional homogeneity of equations—

(i) $s = ut + \frac{1}{2} at^2$

(ii) $S_n = u + \frac{a}{2}(2n-1)$

20. In vander Wall's gas equation $\left(P + \frac{a}{v^2}\right)(v-b)$ = RT. Determine the dimensions of a and b.

21. Using dimensions convert (a) 1 newton into dynes (b) 1 erg into joules.

22. Magnitude of force experienced by an object moving with speed v is given by $F = kv^2$. Find dimensions of k.

23. A book with printing error contains four different formulae for displacement. Choose the correct formula/formulae

(a) $y = a \sin \frac{2\pi}{T} t$ (b) $y = a \sin vt$

(c) $y = \frac{a}{T} \sin\left(\frac{t}{a}\right)$

(d) $y = \frac{a}{T}\left(\sin \frac{2\pi}{T} t + \cos \frac{2\pi}{T} t\right)$

24. Give limitations of dimensional analysis.

25. For determination of 'g' using simple pendulum, measurements of length and time period are required. Error in the measurement of which quantity will have larger effect on the value of 'g' thus obtained. What is done to minimise this error?

SHORT ANSWER QUESTIONS (3 MARKS)

26. Give the name of six Indian Scientists and their discoveries.

27. Name the discoveries made by the following scientists :

(a) Faraday (b) Chadwick

(c) Hubble (d) Maxwell

(e) Newton (f) Bohr.

28. Name the scientific principle on which the following technology is based.

(i) Steam engine (ii) Laser

(iii) Aeroplane (iv) Rocket propulsion

(v) Radio and T.V.

(vi) Production of Ultra high magnetic field.

29. Describe a method for measuring the molecular size of Oleic acid.

[3 MARKS]

30. Describe the Parallox Method for the determination of the distance of a nearby star from the earth.

31. Deduce the dimensional formula for the following quantities

(i) Gravitational constant (ii) Young's modulus

(iii) Coefficient of viscosity.

32. Define the following units :

(i) Light year (ii) Parsec

(iii) Astronomical unit (Au)

LONG ANSWER QUESTIONS (5 MARKS)

33. Name the four basic forces in nature. Write a brief note of each. Hence compare their strengths and ranges.

34. Distinguish between the terms precision and accuracy of a measurement.

35. Explain

(i) absolute error (ii) mean absolute error

(iii) relative error (iv) percentage error

(v) random error

NUMERICALS

36. Determine the number of light years in one metre.

37. The sides of a rectangle are (10..5 ± 0.2) cm and (5.2 ± 0.1) cm. Calculate its perimeter with error limits.

38. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces 20.15 *g* and 20.17 *g* are added to the box.

(i) What is the total mass of the box ?

(ii) The difference in masses of the pieces to correct significant figures.

39. 5.74 *g* of a substance occupies 1.2 cm^3. Express its density to correct significant figures.

40. If displacement of a body $s = (200 \pm 5)\ m$ and time taken by it $t = (20 + 0.2)\ s$, then find the percentage error in the calculation of velocity.

41. If the error in measurement of mass of a body be 3% and in the measurement of velocity be 2%. What will be maximum possible error in calculation of kinetic energy.

42. The length of a rod as measured in an experiment was found to be 2.48m, 2.46m, 2.49m, 2.50m and 2.48m. Find the average length, absolute error and percentage error. Express the result with error limit.

43. A physical quantity is measured as Q = (2.1 ± 0.5) units. Calculate the percentage error in (1) Q^2 (2) 2Q.

44. When the planet Jupiter is at a distance of 824.7 million km from the earth, its angular diameter is measured to be 35.72″ of arc. Calculate diameter of Jupiter.

45. A laser light beamed at the moon takes 2.56*s* and to return after reflection at the moon's surface. What will be the radius of lunar orbit.

46. Convert

(i) $3\ m.s^{-2}$ to km h^{-2}

(ii) $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$ to $cm^3\ g^{-1}\ s^{-2}$

47. A calorie is a unit of heat or energy and it equals 4.2 J where 1J = 1 kg m^2s^{-2}. Suppose we employ a system of units in which unit of mass is α kg, unit of length is $\beta\ m$, unit of time γs. What will be magnitude of calorie in terms of this new system.

48. The escape velocity *v* of a body depends on–

(i) the acceleration due to gravity '*g*' of the planet,

(ii) the radius R of the planet. Establish dimensionally the relation for the escape velocity.

49. The frequency of vibration of a string depends of on, (i) tension in the string (ii) mass per unit length of string, (iii) vibrating length of the string. Establish dimensionally the relation for frequency.

50. One mole of an ideal gas at STP occupies 22.4 L. What is the ratio of molar volume to atomic volume of a mole of hydrogen? Why is the ratio so large. Take radius of hydrogen molecule to be 1ºA.

51. Derive an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r.

52. The error in the measurement of radius of a sphere is 2%. What would be the error in :

(a) Volume of sphere

(b) Surface area of sphere.

SOLUTIONS

1. Speed of high in vacuum, c = 1 new unit of length s^{-1}

t = 8 min. 20 sec, = 500 s

$x = ct$ = 1 new unit of length s^{-1} × 500s

x = 500 new unit of length

2. The unit of left hand side is metre so the units of ct^2 should also be metre. Since t^2 has unit of s^2, so the unit of c is m/s^2.

3. mN means milli newton, 1 mN = 10^{-3} N, Nm means Newton meter, nm means nano meter.

4. $$\frac{V_{atom}}{V_{nucleus}} = \frac{4\pi(10^{-10}m)^3}{4\pi(10^{-15}m)^3} = 10^{15}$$

5. $1u = 1.66 \times 10^{-27}$ kg

6. Work, energy and torque.

7. Stress, pressure, modulus of elasticity.

8. Strain, refractive index.

9. (i)1, (ii) 3, (iii) 4, (iv) 4, (v) 3, (vi) 5 since it comes from a measurement the last two zeros become significant.

10. 2%.

11. Relative error in P is given by

$$\frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\cdot\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

So, percentage error

$$\frac{\Delta P}{P}\times 100 = 3\left(\frac{\Delta a}{a}\times 100\right)+2\left(\frac{\Delta b}{b}\times 100\right)+\frac{1}{2}\left(\frac{\Delta c}{c}\times 100\right)+\frac{\Delta d}{d}\times 100$$

$$= (3 \times 1\%) + (2 \times 3\%) + \left(\frac{1}{2}\times 4\%\right) + (1 \times 2\%)$$

$$= 13\%$$

Rounded off value of P = 3.8.

12. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from 1 but v^2/c^2 can be subtracted from 1.

$$\therefore \qquad m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

13. (a) SONAR $\rightarrow$ Sound Navigation and Ranging.

(b) RADAR $\rightarrow$ Radio Detection and Ranging.

14. (i) Boltzmann Constant :

$$k = \frac{\text{Heat}}{\text{Temperature}} \Rightarrow [k] = \frac{ML^2T^{-2}}{K} = [M^1L^2T^{-2}K^{-1}]$$

(ii) $$[J] = \left[\frac{\text{Work}}{\text{Heat}}\right] = \frac{M^1L^2T^{-2}}{M^1L^2T^{-2}} = [M^0L^0T^0]$$

15. Dimensional Constants : Gravitational constant, plank's constant.

Dimensionless Constants : π, e.

16. Minimum inaccuracy = Vernier constant

$$= 1 \text{ MSD} - 1 \text{ VS.D}$$

$$= 1 \text{ MSD} - \frac{49}{50} \text{ MSD}$$

$$= \frac{1}{50}(0.5 \text{ mm}) = 0.01 \text{ mm}$$

17. $$[F] = [MLT^{-2}]$$

$$[M] = \frac{[F]}{[L][T^{-2}]} = \frac{[100N]}{[10m][100s]^{-2}} = 10^5\text{kg.}$$

19. (i) Dimension of L.H.S. = [s] = $[M^0L^1T^0]$

Dimension of R.H.S. = $[ut] + [at^2]$

$$= [LT^{-1}.T] + [M^0L^1T^{-2}.T^2] = [M^0L^1T^0]$$

as Dimensions of L.H.S. = Dimensions of R.H.S.

$\therefore$ The equation to dimensionally homogeneous.

(ii) S_n = Distance travelled in n^{th} sec that is $(S_n - S_{n-1})$

$\therefore$
$$S_n = u \times 1 + \frac{a}{2}(2n-1)$$
$$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$$
$$[LT^{-1}] = [LT^{-1}]$$
$$\text{L.H.S.} = \text{R.H.S.}$$

Hence this is dimensionally correct.

20. Since dimensionally similar quantities can only be added

$\therefore$
$$[P] = \left[\frac{a}{V^2}\right] \Rightarrow [a] = [PV^2] = [M^1L^5T^{-2}]$$
$$[b] = [v] = [L^3].$$

22. $[K] = \dfrac{[F]}{[v^2]} = \dfrac{M^1L^1T^{-2}}{[LT^{-1}]^2} = \dfrac{M^1L^1T^{-2}}{M^0L^2T^{-2}} = [M^1L^{-1}]$

23. The argument of sine and cosine function must be dimensionless so (*a*) is the probable correct formula. Since

(a) $y = a \sin\left(\dfrac{2\pi}{T}t\right), \therefore \left[\dfrac{2\pi t}{T}\right] = [T^0]$ is dimensionless.

(b) $y = a \sin vt$, $\because$ $[vt] = [L]$ is dimensional so this equation is incorrect.

(c) $y = \dfrac{a}{t}\sin\left(\dfrac{t}{a}\right), \because \left[\dfrac{t}{a}\right]$ is dimensional so this is incorrect.

(d) $y = \dfrac{a}{t}\left(\sin\dfrac{2\pi}{T}t + \cos\dfrac{2\pi t}{T}\right)$: Though $\dfrac{2\pi t}{T}$ dimensionless $\dfrac{a}{T}$ does not have dimensions of displacement so this is also incorrect.

(NUMERICAL)

36. 1 I$y = 9.46 \times 10^{15}$ m

$1\ m = \dfrac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16}$ Iy

37. $P = 2\,(l + b) \pm 2\,(\Delta l + \Delta b)$

$= 2(10.5 + 5.2) \pm 2(0.2 + 0.1)$

$= (31.4 \pm 0.6)$ cm.

38. (i) Mass of box = 2.3 kg

Mass of gold pieces = 20.15 + 20.17 = 40.32 g = 0.04032 kg.

Total mass = 2.3 + 0.04032 = 2.34032 kg

In correct significant figure mass = 2.3 kg (as least decimal)

(ii) Difference in mass of gold pieces = 0.02 g

In correct significant figure (2 significant fig. minimum decimal) will be 0.02 g.

39. Density = $\frac{\text{Mass}}{\text{Volume}} = \frac{5.74}{1.2}$ = 4.783g/cm^3

Here least significant figure is 2, so density = 4.8 g/cm^3.

40. Percentage error in measurement of displacement = $\frac{5}{200} \times 100$

Percentage error in measurement of time = $\frac{0.2}{20} \times 100$

∴ Maximum permissible error = 2.5 + 1 = 3.5%

41. K.E. = $\frac{1}{2}mv^2$

$$\therefore \frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{3\Delta v}{v} \Rightarrow \frac{\Delta k}{k} \times 100 = \frac{\Delta m}{m} \times 100 + 2\left(\frac{\Delta v}{v}\right) \times 100$$

∴ Percentage error in K.E. = 3% + 2 × 2% = 7%

42. Average length

$$= \frac{2.48 + 2.46 + 2.49 + 2.50 + 2.48}{5} = \frac{12.41}{5} = 2.48\text{m}$$

Mean absolute error

$$= \frac{0.00 + 0.02 + 0.01 + 0.02 + 0.00}{5} = \frac{0.05}{5} = 0.013\text{m}$$

Percentage error = $\frac{0.01}{2.48}$ × 100% = 0.04 × 100%

= 0.40%

Correct length= (2.48 ± 0.01)m

Correct length= (2.48m ± 0.40%)

43. P = Q^2

$$\frac{\Delta p}{p} = \frac{2\Delta Q}{Q} = 2\left(\frac{0.5}{2.1}\right) = \frac{1.0}{2.1} = 0.476$$

$$\frac{\Delta p}{p} \times 100\% = 47.6\% = 48\%$$

$$R = 2Q$$

$$\frac{\Delta R}{R} = \frac{\Delta Q}{Q} \Rightarrow \frac{0.5}{2.1} = 0.238$$

$$\frac{\Delta R}{R} \times 100\% = 24\%$$

44. $\theta = 35.72''$

$1'' = 4.85 \times 10^{-6}$ radian $\Rightarrow 35.72'' = 35.72 \times 4.85 \times 10^{-6}$ rad.

$$d = DQ = 824.7 \times 35.72 \times 4.85 \times 10^{-6}$$

$$= 1.4287 \times 10^{5} \text{ km}$$

45. $t = 2.56\ s$

$\therefore$ t = time taken by laser beam to go to the moon = $\frac{t}{2}$

distance between earth and moon

$$= d = c \times \frac{t}{2}$$

$$= 3 \times 10^{8} \times \frac{2.56}{2}$$

$$= 3.84 \times 10^{8} \text{ m.}$$

46. (i) $3\text{m s}^{-2} = \left(\frac{3}{1000}\text{km}\right)\left(\frac{1}{60\times60}\text{hr}\right)^{-2}$

$$= \frac{3\times(60\times60)^2}{1000} = 3.8880 \times 10^{4} \text{ km h}^{-2} = 3.9 \times 10^{4} \text{ km h}^{-2}$$

(ii) $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$

$= 6.67 \times 10^{-11}\ (\text{kg m s}^{-2})\ (\text{m}^2\ \text{kg}^{-2})$

$= 6.67 \times 10^{-11}\ \text{kg}^{-1}\ \text{m}^3\ \text{s}^{-2}$

$= 6.67 \times 10^{-11}\ (1000\ \text{g})^{-1}\ (100\ \text{cm})^3\ (\text{s}^{-2})$

$= 6.67 \times 10^{-11} \times \frac{1}{1000} \times 100 \times 100 \times 100\ \text{g}^{-1}\ \text{cm}^3\ \text{s}^{-2}$

$= 6.67 \times 10^{-8}\ \text{g}^{-1}\ \text{cm}^3\ \text{s}^{-2}$.

47. $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T^2}\right)^c$$

$$= 4.2 \left(\frac{\text{kg}}{\alpha\text{kg}}\right)^1 \left(\frac{\text{m}}{\beta\text{m}}\right)^2 \left(\frac{s}{\gamma s}\right)^{-2}$$

$$n_2 = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$$

48. $v\ \alpha\ g^a\ R^b \Rightarrow v = k\ g^a\ R^b$, K $\rightarrow$ dimensionless proportionality constant

$$[v] = [g]^a\ [R]^b$$

$$[M^0L^1T^{-1}] = [M^0L^1T^{-2}]^a\ [M^0\ L^1\ T^0]^b$$

equating powers

$$1 = a + b$$

$$-1 = -2a \Rightarrow a = \frac{1}{2}$$

$$b = 1 - a = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad v = k\sqrt{gR}$$

49. $n \propto I^a\ T^b\ m^c$, $[I] = M^0L^1T^0$

$$[T] = M^1\ L^1\ T^{-2} \text{ (force)}$$

$$[M] = M^1\ L^{-1}\ T^0$$

$$[M^0\ L^0\ T^{-1}] = [M^0\ L^1\ T^0]^a\ [M^1\ L^1\ T^{-2}]^b\ [M^1\ L^{-1}\ T^0]^c$$

$$b + c = 0$$

$$a + b - c = 0$$

$$-2b = -1 \Rightarrow b = \frac{1}{2}$$

$$c = -\frac{1}{2}\ a = 1$$

$$n \propto \frac{1}{\ell}\sqrt{\frac{T}{m}}.$$

50. $1\ A^0 = 10^{-10}$ m

Atomic volume of 1 mole of hydrogen

= Avagadros number × volume of hydrogen molecule

$$= 6.023 \times 10^{23} \times \frac{4}{3} \times \pi \times (10^{-10}\ \text{m})^3$$

$$= 25.2 \times 10^{-7}\ \text{m}^3$$

Molar volume $= 22.4 \text{ L} = 22.4 \times 10^{-3} \text{ m}^3$

$$\frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}}{25.2 \times 10^{7}} = 0.89 \times 10^4 \approx 10^4$$

This ratio is large because actual size of gas molecule is negligible in comparison to the inter molecular separation.

❑❑